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Capital Income Taxation with Portfolio Choice

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Capital Income Taxation with Portfolio Choice

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Abstract:

This paper analyzes redistributional and macroeconomic effects of differential taxation of financial assets with a different risk levels. The redistributive effect stems from the fact that various households hold portfolios with a starkly different risk levels. In particular, poor households primarily save in safe assets, while rich households often invest a substantially higher share of their wealth in (risky) equity. At the same time, equity and safe assets are often taxed at different rates in many tax codes. This is primarily because investments in equity (which are relatively riskier) are taxed both as corporate and personal income, unlike debt, which is tax deductible for corporations.
This paper firstly builds a simple theoretical two-period model, which shows that the optimal tax wedge between risky and safe assets is increasing in the underlying wealth inequality.
Furthermore, I build a quantitative model with a continuum of heterogeneous agents, parsimonious life-cycle, borrowing constraint, aggregate shocks and uninsurable idiosyncratic shocks, in which the government raises revenue by using linear taxes on risky and safe assets. Simulations of quantitative models shows that elimination of differential asset taxation leads to a welfare loss equivalent to a 0.3 % permanent reduction in consumption. I find that the optimal tax wedge between taxes on equity and debt is higher than the one in the U.S. tax code.

Zusammenfassung:

Dieses Papier analysiert die Umverteilungs- und makroökonomischen Auswirkungen einer differenziellen Besteuerung von Finanzanlagen mit unterschiedlicher Risikobehaftung. Der Umverteilungseffekt ergibt sich daraus, dass sich Haushalte stark in der Risikobehaftung ihrer Portfolios unterscheiden. Insbesondere arme Haushalte wählen sichere Anlageformen für ihre Ersparnisse, während reiche Haushalte oft einen wesentlich höheren Anteil ihres Vermögens in (risikoreiche) Aktien investieren. Gleichzeitig werden Aktien und sichere Vermögenswerte in vielen Steuergesetzen oft zu unterschiedlichen Sätzen besteuert. Dies liegt in erster Linie daran, dass Investitionen in Aktien (die relativ risikoreicher sind) sowohl als Unternehmens- als auch als Privateinkommen besteuert werden, im Gegensatz zu Schulden, die für Unternehmen steuerlich absetzbar sind. In diesem Beitrag wird zunächst ein einfaches theoretisches Zwei-Perioden-Modell vorgestellt, das zeigt, dass der optimale Steuerkeil zwischen risikoreichen und sicheren Vermögenswerten mit der zugrunde liegenden Vermögensungleichheit zunimmt. Darüber hinaus entwickle ich ein quantitatives Modell mit einem Kontinuum heterogener Agenten, sparsamem Lebenszyklus, Kreditaufnahmebeschränkungen, aggregierten Schocks und nicht versicherbaren idiosynkratischen Schocks, in dem der Staat die Einnahmen durch den Einsatz linearer Steuern auf riskante und sichere Vermögenswerte erhöht. Simulationen quantitativer Modelle zeigen, dass die Abschaffung der differenziellen Vermögensbesteuerung zu einem Wohlfahrtsverlust führt, der einem dauerhaften Rückgang des Konsums um 0,3 % entspricht. Der optimale Steuerkeil zwischen Steuern auf Eigen- und Fremdkapital liegt höher als der im US-Steuerrecht.

Keywords:

Portfolio Choice, Optimal Taxation, Redistribution

JEL Classification:

E62, G11, G32, H21, H23

Capital Income Taxation with Portfolio Choice*

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Abstract

This paper analyzes redistributional and macroeconomic effects of differential taxation of financial assets with a different risk levels. The redistributive effect stems from the fact that various households hold portfolios with a starkly different risk levels. In particular, poor households primarily save in safe assets, while rich households often invest a substantially higher share of their wealth in (risky) equity. At the same time, equity and safe assets are often taxed at different rates in many tax codes. This is primarily because investments in equity (which are relatively riskier) are taxed both as corporate and personal income, unlike debt, which is tax deductible for corporations. This paper firstly builds a simple theoretical two-period model which shows that the optimal tax wedge between risky and safe assets is increasing in the underlying wealth inequality. Furthermore, I build a quantitative model with a continuum of heterogeneous agents, parsimonious life-cycle, borrowing constraint, aggregate shocks and uninsurable idiosyncratic shocks, in which the government raises revenue by using linear taxes on risky and safe assets. Simulations of quantitative models shows that elimination of differential asset taxation leads to a welfare loss equivalent to a 0.3% permanent reduction in consumption. I find that the optimal tax wedge between taxes on equity and debt is higher than the one in the U.S. tax code.

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1 Introduction

Traditionally, the optimal taxation literature has focused on the optimal taxation of capital, including studies such as Chamley (1986), Judd (1985), Straub and Werning (2020), Chari et al. (2018), and many others. The literature has so far usually assumed that the capital is homogeneous. However, financial capital has two main tranches: debt and equity. Since debt is the highest priority tranche, it is usually less risky than equity.[1] These tranches are usually taxed at different rates and subject to different types of taxes (for example, debt and interest are not subject to corporate income tax). Furthermore, empirically, different types of households allocate their wealth between these two tranches in starkly different proportions. Consequently, non-uniform taxation of these two tranches of capital leads to redistribution between different types of households. This paper analyzes, normatively and positively, the effects of such differential taxation, focusing on its redistributionary dimension.

Survey data reveal the heterogeneity of the portfolio structure for households of different wealth. One of the most relevant surveys that shows this is the Survey of Consumer Finances (SCF), which collects data on US. households. Rich households are found to save disproportionately more in risky assets compared to poor households. The non-participation in financial markets by some poor households cannot alone explain this feature, because the share of wealth held in risky assets, conditional that a household participates in the financial market, is also increasing. The pattern that rich households invest more in risky assets is present even when controlling for the age of the households.

Significant portfolio differences between rich and poor households potentially has important policy implications for a government that maximizes utilitarian social welfare. More precisely, a utilitarian government might want to tax the assets in which the rich households save (risky asset) relatively heavily, in order to be able to tax the asset in which poor households save (safe assets) if the usual concave utility function is assumed. This feature of the tax code is widespread in developed countries in the form of double taxation of returns from equity (dividends).

[1] Abstracting from the risk of inflation.

At the same time, there is an ongoing debate about the double taxation of returns from equity. This feature is often interpreted as a higher tax on risky capital income (equity), compared to a safe asset (debt) (see Scheuer, 2013). It is often argued that this feature of the tax code has a negative impact on welfare, as it distorts the financing sources of firms, and therefore induces excessively large leverage. Consequently, in recent decades, some countries have attempted to reduce the difference between the two effective tax rates (for example, the "Bush tax reform" in 2003, with the introduction of qualified dividend, which was extended by the Obama administration in 2013). This is achieved both by reducing capital income tax, and by taxing the dividend payouts at the reduced rate in the personal income tax. However, the reduced tax rate for dividends in the personal income tax code is relevant only if the households hold the equity directly and not through pension funds (which is the way through which the majority of poor and middle-class people hold equity, see Rios-Rull and Kuhn (2016)).

This research takes a different, complementary perspective on the question of optimal taxation of debt and equity. Instead of focusing on the issuers of the assets (firms), it focuses on the holders of the assets (households). More precisely, it analyzes the redistributionary effects of a distortionary policy which can shift the tax burden from poor to rich households. In other words, differential taxation can make insurance (precautionary savings in the safe asset) cheaper for poor households who need (and use) it the most. Therefore, given the heterogeneity of portfolios of rich and poor households, the social planner might find differential taxation of capital income optimal, even if it is associated with certain efficiency costs.

This paper does not examine the normative question of whether capital income tax is optimal as such. Instead, it seeks to contribute to the literature by attempting to answer the following questions: given that capital is taxed, can the differential tax treatment of income from assets of a different risk be optimal, and what are the redistributional consequences of such a policy? Note that this question can be asked even if the average tax on the overall capital is zero. To the best of my knowledge, the redistributional effects of the differential financial capital taxation have not yet been analyzed. In addition to the redistributive consequences, I examine the effects on the aggregate savings rate.

Therefore, this paper has two dimensions: first, the normative one is to examine whether it is optimal to tax different types of capital differently. Second, the positive dimension attempts to quantify the effects of differential taxation of capital in the US.

The paper analyzes the importance of differential taxation in a dynamic quantitative macroeconomic model. The model features endogenous portfolio choice, a continuum of heterogeneous agents, uninsurable idiosyncratic risk, and aggregate risk. Furthermore, the model is calibrated in attempt to match the wealth and earnings distribution in the US, as well as the generating substantial equity premium and the portfolio choice patterns. The model includes many features that break the uniform taxation result, such as uninsurable idiosyncratic labor income risk, which is correlated with the returns of the risky asset, borrowing and subsistance constraints, and parsimonious life-cycle.

The logic from the Ramsey taxation framework is that the utilitarian social planner can find it optimal to tax a risky asset at a higher rate for redistributionary motives, even if the differential taxation is associated with efficiency costs. The two-period Ramsey taxation model is described in Section 3. Numerical simulations of the full-blown model show that poor households indeed prefer the relatively low taxation of safe assets, and wealthy households prefer relatively lower taxation of risky assets. This is because the households' portfolios, greatly governed by their exposure to labor income risk, are relatively skewed towards risky in rich and safe in poor households. However, taxing the risky asset at a relatively lower rate can promote capital accumulation, which consequently raises wages and decreases interest rates. As poorer households rely more on labor income, and rich households on capital income, these general equilibrium effects tend to reduce the inequality. Finally, it can also be interesting to consider the examined mechanisms in the context of Davila et al. (2012), who find that in these types of models, there is a severe under-accumulation of capital, compared to the constrained efficient outcome. A higher wedge between taxes on equity and debt slightly decreases capital accumulation and decreases the insurance of the relatively poor households. On the other hand however, it increases their possibilities of insurance, as the returns on safe assets, in which they primarily save, increase. Furthermore, heavy taxation of risky equity can be beneficial even for wealthy households, as it reduces the variance of the returns of the risky asset (the taxes are high when the returns are high, but are low when the returns are low).

The remainder of this paper is organized as follows: Section 2 reviews the existing literature, Section 3 builds an analytical model and solves for the optimal tax formula. Section 4 describes the benchmark quantitative model. Section 5 describes the performed numerical exercises and presents the results. Finally, the paper concludes.

2 Literature review

The topics of redistributive and capital income taxation have attracted much interest from economists. The famous Chamley-Judd result of zero long-run capital taxation (Chamley, 1986; Judd, 1985) was shown by Straub and Werning (2020) not to hold generally even in the models from which it was derived. Many other papers have shown that capital taxation can be optimal in a life cycle model if the government (as is usually the case in the majority of countries), can not condition taxes on the age of a household: Erosa and Gervais (2002), Conesa et al. (2009). Furthermore, Panousi and Reis (2012) show that capital income tax, combined with other policy instruments, can increase the aggregate capital accumulation because it reduces the variance of the investment returns for entrepreneurs. In addition, Saez (2013) studies optimal progressive capital income taxes in an infinite horizon model where agents differ only in their initial wealth. Chari et al. (2018) revisit the results of Straub and Werning (2020), arguing that when one abstracts from the expropriation of the initial capital, zero capital taxation result reemerges in the basic models.

This paper does not examine the normative question of whether capital income tax is optimal as such, but it poses the question: can the differential tax treatment of income from assets of different risk be optimal, and what are the redistributional consequences of such policy? To the best of the author's knowledge, the redistributional effects of the differential financial capital taxation have not yet been explored. In addition to the main question, other effects of differential capital income taxation on the economy, such as effects on saving rates and asset prices, will be examined as well.

The second related strand of literature examines the differential financial asset taxation. Notable papers that have recently examined the desirability of the differential financial asset taxation from the efficiency perspective are Ferris (2018), which looks at its effect on stock volatility, and Chetty and Saez (2010), which attempts to develop an empirically implementable formula for the efficiency cost of dividend taxation. They both find a significant efficiency cost of differential asset taxation. However, they do not consider the setup with heterogeneous households. There are important results regarding taxation and entrepreneur portfolio choices in the Mirrleesian, private information framework (Mirrlees, 1971), e.g. papers like Shourideh (2012) and Albanesi (2011) consider the optimal Mirrleesian taxation problem of entrepreneurial income. The results are therefore relevant, but they crucially differ from this paper because they consider the problem of taxing the entrepreneurs, who have private information about their businesses, while my analysis focuses on portfolio choices of agents who seek to allocate their savings, and are not necessarily entrepreneurs. Shourideh (2012) studies the optimal taxation of wealthy individuals in the Mirrlessian environment in which the sources of inequality are capital income shocks and financial frictions. He finds that the optimal tax schedule is characterized by progressive savings tax and negative bequest tax. Albanesi (2011) studies optimal taxation of entrepreneurial capital in which entrepreneurs have private information, and entrepreneurial activity is thus subject to moral hazard. The main results of the model are that the differential asset taxation and double taxation of capital income are found to be optimal. Another paper with relevant results in the Mirrleesian tradition is Scheuer (2013). He constructs a highly abstract model with aggregate uncertainty, where heterogeneous agents trade consumption claims contingent on aggregate shocks in financial markets. The main feature of the optimal tax code is that optimal asset taxes are higher for the securities that payout in aggregate states where consumption is more volatile. This result is compatible with the result of the quantitative part of this paper. He argues that this can provide a theoretical efficiency justification for the differential tax treatment of different asset classes (for example, debt and equity). However, he does not consider the redistributional effects of such differential taxation, which will be of crucial interest in my research.

Unlike in the above models in the Mirrlessian tradition, differential taxation of assets is often found not to be the "first best", meaning that it would be optimal to transfer across different agents, without distorting the incentives in the economy. However, such taxes would be highly complex and almost certainly unimplementable by the government (for example, tax evasion, or simple unconstitutionality of laws that condition taxes on the age of a person). Therefore, governments often have to rely on the more straightforward (for example, linear) taxes, and in this context differential asset taxation can be found to be optimal, often referred to as the second best.

Analysis by Slavík and Yazici (2014) is an example of an arguably desirable differential capital taxation because of its heterogeneous effects in the population, although in a different context. They find that differential taxation of different types of physical capital can be beneficial because it can promote investments in types of physical capital that are complementary with a low skilled type of labor, which benefits the poor agents.

In this paper, the redistribution channel comes from the stylized fact that poor households save mainly in a safe asset, while wealthy households invest mostly in risky assets. This has been well documented in US micro survey data (Survey of Consumer Finances) by Guiso et al. (2000), Poterba and Samwick (2003) and Chang et al. (2018). For example, Chang et al. (2018) document that, according to Survey of Consumer Finances, in the US in 1998, the poorest households (1st quintile of the wealth distribution), invested 5.3% of their wealth in risky assets, while the richest households (5th quintile of the wealth distribution) held 64.9% of their wealth in risky assets. They also show that the positive correlation of wealth and the share of wealth invested in the risky asset cannot be solely explained by the non-participation of poor households in the equity markets. The reasoning behind the fact that a wealthy household invests a larger share of their wealth in a risky asset is illustrated in the following way: wealthy agents do so because they have big capital stock to act as a buffer against the idiosyncratic risk of their future labor income, which allows them to bear more risk in the financial markets.[2] Importantly, it is not easily observed in the data what portion of the assets is invested in risky as opposed to safe assets. This is because most households (except perhaps the wealthy), hold a considerable amount of their wealth in mutual funds and retirement accounts (Rios-Rull

[2]See Algan et al. (2009) and Chang et al. (2018).

and Kuhn, 2016). However, SCF data provide additional information showing where these funds are eventually invested (equity or safer assets). Chang et al. (2018) use this information to calculate the risky/safe split of the household's wealth.

Guiso et al. (2000), Chang et al. (2018) also notice that, in the data, the share of wealth invested in risky assets is increasing with the age of the household. This stylized fact can also motivate the inclusion of the life-cycle dimension into the question (disproportionately taxing risky asset means disproportionately taxing older households). This can as well provide a reason for the optimality of differential asset taxation, because (in the absence of age-dependent taxes) it provides another policy instrument that can distinguish between young and old agents. This would enable a government to differentiate between agents of various ages much more finely, compared to the case in which it relies only on having positive (uniform) capital income tax (which is found to be optimal by Erosa and Gervais (2002), in the context of non-age dependent labor taxes).[3]

Studies such as Poterba and Samwick (2003) show that the effect of the tax structure (a particular implementation of tax systems, for example, corporate and personal income tax) impacts the portfolio choice. They find that households with a high marginal personal income tax rate prefer to hold their wealth in stocks as opposed to interest-bearing assets because, in the US, most equity is taxed at the reduced *personal* income tax rate (though it is taxed by corporate tax as well). Unlike their study, this paper abstracts from the issues of different tax instruments for the sake of simplicity and computational feasibility.

In addition to examining the distributional effects of differential asset taxation, this project will also study its effects on the aggregate savings rate in the economy. In particular, the presumed progressive nature of differential asset taxation can potentially increase the precautionary savings motive of the wealthy agents, increasing their savings rate.

[3] See also Conesa et al. (2009).

3 Two-period Ramsey model

A simple two-period model is presented to develop the intuition and obtain insights from a minimal working example. Two agents differ only in their initial wealth m^h.

The setup is a standard Ramsey problem in which the agents invest in two types of capital (instead of traditionally in consumption goods). Let us say that the agent has the following preferences

$$u(c_h) = \frac{(c_h + L)^{1-\alpha}}{1-\alpha}$$

where c_h is the consumption of agent h, L is the exogenous labor income (if it is negative, it can be interpreted as a subsistence level of consumption), and α is a parameter (coefficient of risk aversion). The preferences are standard constant relative risk aversion (CRRA) preferences, with the addition of L (exogenous labor income or subsistence level).

Furthermore, assume that there are two possible states of the world: *"Good"* and *"Bad"*, and that prior to the realization of these states, agents have to decide how to allocate their wealth m_h between two assets: 1) a safe asset that pays off 1 unit of consumption good in both good and bad states, and 2) a risky asset that pays of r_g in a good state and r_b in a bad state of the world ($r_g > 1$, $r_b < 1$, and $\frac{r_g + r_b}{2} > 1$, and technologies are linear). Thus, the consumer maximizes the expected utility:

$$E(u(c_h)) = \frac{1}{2}\frac{(c_g^h + L_g)^{1-\alpha}}{1-\alpha} + \frac{1}{2}\frac{(c_b^h + L_b)^{1-\alpha}}{1-\alpha}$$

where

$$c_g^h = R^h r_g + S^h$$

$$c_b^h = R^h r_b + S^h$$

subject to the constraint $R^h q_r + S^h q_s = m^h$, where q_r and q_s are (after-tax) prices.

Considering this setup, it is possible to rewrite the consumer utility function as a function of two goods: risky asset R, and safe asset S:

$$U(R^h, S^h) = \frac{1}{2}\Big\{\frac{(R^h r_g + S^h + L_g)^{1-\alpha}}{1-\alpha} + \frac{(R^h r_b + S^h + L_b)^{1-\alpha}}{1-\alpha}\Big\}$$

For simplicity, it is assumed that $L_b = 0$. Now, we can set up a Ramsey problem in which the government has to tax the two goods (R and S) to raise some exogenously given revenue G.

The drawback of this approach is that the government is taxing the agents when they are buying the assets and is not taxing the returns of the assets (which is more realistic). However, the main results extend to the case in which the government is taxing the returns on the two assets. This is verified numerically in Appendix B. The advantage of this approach is that we can use the existing results of taxation theory and compare our results to other standard Ramsey models. In addition, this approach removes the questions of debt and state-dependent taxes for the sake of simplicity.

Now consider the Ramsey problem by the government:

$$\max_{\tau_r, \tau_s} W = \sum^{h} U(c^h)$$

subject to:

$$\tau_r \sum^{h} R^h + \tau_s \sum^{h} S^h \geq G$$

and agents' maximizing decisions.

The questions being asked are: should there be differential taxation of the two assets, in which cases, and in what direction?

Ramsey problem

The government sets τ_r and τ_s to raise the revenue G, and achieve the highest welfare possible for the agents, anticipating their optimal decisions.

Notation:

-let $x_j^h(p, m^h)$ be the Marshalian demand for good j of agent h

-let $V^h(p, m^h)$ be the agent's indirect utility function

-let $\hat{x}_j^h(p, m^h)$ be the compensated (Hicksian) demand for good j

-let $\chi(p, m)$ be the utilitarian social welfare function equal to $\sum^h V^h(p, m^h)$

-let $p_j = (1 + \tau_j)$ be the after tax price of asset j

-let λ be a lagrangian multiplier on a government constraint

Using this notation, the Ramsey problem is:

$$\max_{\tau_r \tau_s} \chi(p, m)$$

s.t.

$$\tau_r(\sum_{h=1}^{2} x_r^h(p, m^h)) + \tau_s(\sum_{h=1}^{2} x_s^h(p, m^h)) \geq G$$

Furthermore, following notation is used:

$$X_k = \sum_{h=1}^{H} x_k^h(p, m)$$

where X_k is the total demand for good k

$$\beta^h = \frac{\frac{\partial \chi}{\partial V^h} \alpha^h}{\lambda} + \sum_{j=1}^{J} \tau_j \frac{\partial x_j^h(p, m^h)}{\partial m}$$

where β^h is the so called "net social marginal utility of income for agent h"

$$\bar{\beta} = \sum_{h=1}^{H} \frac{\beta^h}{H}$$

Now, denote by θ_k the empirical covariance between β^h and $h's$ consumption of good k:

$$\theta_k = cov\Big(\frac{\beta^h}{\bar{\beta}}, \frac{x_k^h}{\bar{X}_k}\Big) = \frac{1}{H} \sum_{h=1}^{H} \Big(\frac{\beta^h}{\bar{\beta}} - 1\Big)\Big(\frac{x_k^h}{\bar{X}_k} - 1\Big)$$

Positive θ_k means that good k is mostly consumed by agents with higher β^h (typically, poor agents).

Lemma 1 *The optimal linear tax policy from the social planner, which maximizes the expected utility of the two agents, satisfies the following equations:*

$$-\frac{\sum_{j=1}^{J} \tau_j p_j (\sum_{h=1}^{H} \frac{\partial \hat{x}_k^h}{\partial p_j})}{X_k} = 1 - \bar{\beta} - \bar{\beta}\theta_k$$

Where $J = 2$ is the number of assets, and $H = 2$ is the number of agents.

Proof. Appendix A.1 ■

This is almost identical to the so-called "a many-person Ramsey tax rule" (Diamond, 1975).

Interpretation of the formula:

The left-hand side is called the "discouragement index for good k under a tax system", and is a measure of a percentage reduction in demand on good k as a consequence of taxation.

The right-hand side is called the "redistributive factor of good k". In a representative agent economy $\theta_k = 0$.

Rewriting the formulas in terms of elasticities of Hicksian demand ($\hat{\epsilon}^h_{i,j} = \frac{p_j}{x_i}\frac{dx^h_k}{dp_j}$):

$$\sum_{j=1}^{J}\left(\frac{\tau_j}{1+\tau_j}\sum_{h=1}^{H}\hat{\epsilon}^h_{k,j}\right) = 1 - \bar{\beta} - \bar{\beta}\theta_k \tag{1}$$

3.1 The case with no labor income

Now, let's consider a case where there is an inequality in the initial wealth, but there is no exogenous labor income in the second period.

Lemma 2 *With no exogenous labor income in the second period, both agents invest the same share of their wealth in the risky asset, and consequently $\theta_r = \theta_s = 0$.*

Proof. Marshalian demand for the risky asset is:

$$x^h_r = \frac{\frac{m^h}{p_s}(A-1)}{r_g - r_b A + \frac{p_r}{p_s}(A-1)}$$

where

$$A = \left(\frac{r_g - \frac{p_r}{p_s}}{\frac{p_r}{p_s} - r_b}\right)^{\frac{1}{\alpha}} > 1$$

Marshalian demand for the safe asset is:

$$x^h_s = \frac{\frac{m^h}{p_r}(Ar_b - r_g)}{1 - A + \frac{p_s}{p_r}(Ar_b - r_g)}$$

Both are linear in m^h.

Computing the income elasticities:

$$\epsilon_{r,m} = \frac{m^h}{x_r^h}\frac{dx_r^h}{dm^h} = 1 = \epsilon_{s,m}$$

■

This is the standard property of CRRA preferences.

Proposition 1 *If there is no exogenous labor income in the second period, the optimal tax schedule taxes both assets at the same rate.*

Proof. Appendix A.2 ■

In this case, the social planner cannot use differential asset taxation to shift the tax burden from one agent to another because all agents will spend an equal share of their wealth on investing in the two assets. In this particular setup, the mentioned investment pattern is a consequence of constant relative risk aversion preferences. In addition, since there are no externalities, in the absence of taxes, the agents choose the optimal investment allocation. Therefore, the social planner should decrease the investment in assets proportionally. This is the standard equal taxation result in the Ramsey taxation literature. Finally, one can see the result in the following way: since the initial wealth is exogenous, the social planner can mimic the (non-distortive) lump-sum tax by taxing all the assets at the same rate.

3.2 The case with exogenous labor income in the second period

The feature that both rich and poor people invest the same share in risky assets is counter-factual. A stylized fact is that the rich invest disproportionately more in risky assets. To generate this pattern, the exogenous labor income in the second period is assumed; $L_g > 0$ in the *good* period and $L_b = 0$ in the*bad* period (where L_b is set to 0 for simplicity). Alternatively, instead of adding labor, it is possible to assume a subsistence constraint also known as The Stone-Geary utility function, which would be equivalent to setting $L_g = L_b < 0$. This means that the agents have exogenous labor income, which is perfectly positively correlated with the returns to risky assets. This feature breaks the homotheticity.[4]

[4] In the quantitative model, both features: subsistance constraint and risky labor income are present, which reinforces the pattern.

Marshalian demands are:[5]

$$x_r^h = \frac{\frac{m^h}{p_s} - L_g(A-1)}{r_g - r_b A + \frac{p_r}{p_s}(A-1)}$$

$$x_s^h = \frac{\frac{m^h}{p_r}(Br_b - r_g) - L_g}{1 - B + \frac{p_s}{p_r}(Br_b - r_g)} = \frac{\frac{m^h}{p_s} - p_r m^h(A-1) + \frac{p_r}{p_s}L_g}{r_g - r_b A + \frac{p_r}{p_s}(A-1)}$$

The introduction of labor income L_g makes the demand for the safe asset less sensitive to changes in income (prices). The reason is that since the agent wants to smooth the consumption across the two states, and in the good state the agent receives L_g, independently of the portfolio choice, the agent primarily wants to buy the asset that has a higher return in the bad state: the safe asset.

Proposition 2 *In the case with exogenous labor income in the second period, if there is no initial inequality: $\forall h : m^h = m$, it is optimal for the social planner to tax both assets at the same rate.*[6]

Proof. Appendix A.3. ■

Agents choose the first best allocation in the absence of taxes. Furthermore, since there is no initial inequality, they will choose exactly the same portfolio, and the social planner does not have any incentive or the tools to shift the tax burden between different agents. A social planner can mimic the lump-sum tax (which is non-distortive) by having equal taxes on both assets. However, if the wealth in the first period was endogenous (for example, if the agents were choosing their labor income), or there were additional investments on which the agents would spend their initial wealth, it would be optimal to tax the safe asset more. This is because the income elasticity of demand for the risky asset is higher than the income elasticity of demand for the safe asset. (see Appendices A.3. and A 5.)

[5] $B = \left(\frac{1 - \frac{p_s}{p_r} r_g}{\frac{p_s}{p_r} r_b - 1}\right)^{\frac{1}{\alpha}}$

[6] If there are more than two assets, or if the labor supply is endogenous, the government can find it optimal to tax the safe asset at a higher rate.

Let us now assume that two agents differ in their initial unearned wealth m^h. In this case, we have an effect from Proposition 2, with an additional, redistributive effect. Since with the introduction of risky labor income the share of wealth invested in risky assets is increasing in wealth, the rich agent will invest disproportionally more in the risky asset compared to the poor agent. Therefore, the government can tax the poor agent less by taxing the safe asset by a lower rate than the risky asset.

Proposition 3 *In the case with exogenous labor income in the second period, a mean preserving spread in M increases the ratio of optimal taxes $\frac{\tau_r}{\tau_s}$.*

Proof. Appendix A.4. ■

This model gives the intuition as to why the government may want to tax the risky assets (for example equity) at a higher rate in order to shift the tax burden from the poor to the relatively rich households. The result that the optimal tax rate $\frac{\tau_r}{\tau_s}$ is increasing in the initial inequality extends to the, more realistic, case in which the government taxes the returns on assets. The example is considered in Appendix B.

This result may seem contradictory to the intuition of the "production efficiency theorem" from Diamond and Mirrlees (1971), which states that the intermediary production goods should not be taxed. However, Diamond and Mirrlees (1971) assume that the government can tax the final goods at different rates, while in the presented model this is not possible because the two assets (intermediary goods) are used to produce the same final good. Therefore, the government cannot distort the after-tax prices of the final goods for the redistribution purposes, and it is therefore forced to use distortionary "intermediate goods" prices. Furthermore, the outcome shows that the result (taxing the safe industry more if the household has decreasing relative risk aversion) from Atkinson and Stiglitz (1972) can be overturned in the presence of heterogeneous agents.

4 Quantitative Model

I construct the model based on (Algan et al., 2009), and in the tradition of Krusell and Smith (1997). The model consists of a continuum of heterogeneous agents facing aggregate risk, uninsurable idiosyncratic labor risk and a borrowing constraint, and who save in two assets: risky equity and safe bonds. Unlike the above-mentioned models, the model parsimoniously captures the life cycle of the households, in the fashion of Castaneda et al. (2003), in which working-age agents face the retirement shock and retired households face the risk of dying. In this model, I introduce the fiscal policy, in which the government uses capital income taxes to finance exogenous government spending G, and labor taxes to finance unemployment benefits, social security (benefits to the retired households), and to balance the budget. The tax rate on the income from bonds (safe asset) is exogenously set to be lower than the tax rate on equity (risky asset) by C^f. To evaluate the effects of differential taxation, the coefficient C^f is varied, and the implications of this variation on the economy are studied. Unlike in the analytical, two-period model, the general equilibrium effects will be present. This is important, firstly because the analysis would be incomplete without considering the (potentially important) welfare implication of such effects. Moreover, the portfolio choice problem is notoriously sensitive, and changing the interest rate (or taxes) in the partial equilibrium even slightly, can cause drastic changes to the optimal portfolio choice. In the general equilibrium model, such responses are mitigated by the price adjustments stemming from the general equilibrium effects.

4.1 Production technology

In each period t, the representative firm uses aggregate capital K_t, and aggregate labor L_t, to produce y units of final good with the aggregate technology $y_t = f(z_t, K_t, L_t)$, where z_t is an aggregate productivity shock. I assume that z_t can take only two values, and it follows a stationary Markov process with transition function $\Pi_t(z, z') = Pr(z_{t+1} = z'|z_t = z)$. The production function is continuously differentiable, strictly increasing, strictly concave and homogeneous of degree one in K and L. Capital depreciates at the stochastic rate $\delta_t \in (0, 1)$ and it accumulates according to the standard law of motion:

$$K_{t+1} = I_t + (1 - \delta_t)K_t$$

where I_t is aggregate investment. The particular aggregate production technology is:

$$Y_t = z_t A K_t^{\gamma} L_t^{1-\gamma}$$

4.2 Preferences

Households are indexed by i and they have identical, recursive preferences, for the retired agents:

$$V_{R,i,t} = \left[c_t^{1-\rho} + v\beta\left[E_t V_{R,i,t+1}^{1-\alpha}\right]^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}$$

where $V_{R,i,t}$ is the recursively defined value function of a retired household i, at time period t.

Working-age agents maximize:

$$V_{W,i,t} = \{c_t^{1-\rho} + \beta\left[(1-\theta)E_t V_{W,i,t+1}^{1-\alpha} + \theta E_t V_{R,i,t+1}^{1-\alpha}\right]^{\frac{1-\rho}{1-\alpha}}\}^{\frac{1}{1-\rho}}$$

4.3 Life cycle structure

In each period, working-age households have a chance of retiring θ, and retired households have a chance of dying v, similarly as in Castaneda et al. (2003) and Krueger et al. (2016). Therefore, the share of working age households in the total population is:

$$\Pi_W = \frac{1-v}{(1-\theta)+(1-v)}$$

and the share of the retired households in the total population is:

$$\Pi_R = \frac{1-\theta}{(1-\theta)+(1-v)}$$

The retired households who die in period t are replaced by new-born agents who start at a working age without any assets. For simplicity, the retired households have perfect annuity markets, which make their returns larger by a fraction of $\frac{1}{v}$, as in Krueger et al. (2016).

4.4 Idiosyncratic uncertainty

In each period, working-age households are subject to an idiosyncratic labor income risk that can be decomposed into two parts. The first part is the employment probability that depends on aggregate risk and is denoted by $e_t \in (0,1)$. $e = 1$ denotes that the agent is employed, and $e = 0$ that the agent is unemployed. Conditional on z_t, z_{t+1} I assume that the period $t+1$ realization of the employment shock follows the Markov process.

$$\Pi_e(z, z', e, e') = Pr(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z')$$

This labor risk structure allows idiosyncratic shocks to be correlated with the aggregate productivity shocks, which is consistent with the data and generates the portfolio choice profile such that the share of wealth invested in risky assets is increasing in wealth. A condition imposed on the transition matrix and the law of large numbers implies that the aggregate employment is only the function of the aggregate productivity shock.

In case that $e = 1$ and the agent is employed, one can assume that the agent is endowed with $l_t \in L \equiv \{l_1, l_2, l_3, ...l_m\}$ efficiency labor units, which she can supply to the firm. Labor efficiency is independent of the aggregate productivity shock, and is governed by the stationary Markov process with transition function $\Pi_l(l, l') = Pr(l_{t+1} = l' | l_t = l)$. If the agent is unemployed, (s)he receives an exogenous amount of final good g_u, which can be interpreted as home production or social insurance.

4.5 The representative firm

As in Algan et al. (2009), firm leverage in my model is given exogenously. The leverage of the firm is determined exogenously by the parameter λ. The Modigliani-Miller theorem (1958, 1963) is rendered invalid by the fact that some of the agents are borrowing constrained. Therefore, the leverage of the firm has macroeconomic relevance.

In the economy, the representative firm can finance its investment with two types of contracts. The first is a one-period risk-free bond that promises to pay a fixed return to the owner. The second is risky equity that entitles the owner to claim the residual profits of the firm after the firm pays out wages and debt from the previous period. Both of these assets are freely traded in competitive financial markets. By construction, there is no default in the equilibrium.

The return on the bond r^b_{t+1} is determined by the clearing of the bond market:

$$\int g^{b,j} d\mu = \lambda K'$$

where $g^{b,j}$ are the individual policy functions for bonds.

Next, the return on the risky equity depends on the realizations of the aggregate shocks and is given by the equation:

$$(1 + r^s_{t+1}) = \frac{f(z_{t+1}, K_{t+1}, L_{t+1}) - f_L(z_{t+1}, K_{t+1}, L_{t+1})L_{t+1} - \lambda K_{t+1}(1 + r^b_{t+1}) + (1 - \delta_{t+1})K_{t+1}}{(1 - \lambda)K_{t+1}}$$

An important caveat in having heterogeneous households that own the firm is that they do not necessarily have the same stochastic discount factor m^j_{t+1}, and therefore the definition of the objective function of the firm is not straightforward. I follow Algan et al. (2009), who assume that the firm is maximizing the welfare of the agents who have interior portfolio choice, and consequently the firm has the same stochastic discount factor m_{t+1}, as the agents with the interior portfolio choice.

Proposition 4 *In equilibrium, the aggregate capital stock K_{t+1} is equal and ex-dividend firm value V_t are equal to the present discounted value of the firm's net cash flows:*

$$K_{t+1} = V_t = E_t \left\{ \sum_{j=1}^{\infty} m^f_{t,t+j} \left[f_K(z_{t+j}, K_{t+j}, L_{t+j})K_{t+j} - I_{t+j} \right] \right\}$$

where $m^f_{t,t+j}$ is the stochastic discount factor of the firm.

Proof. See Algan et al. (2009) ■

This proposition is used to eliminate the capital Euler equation from the equilibrium conditions, and instead use $V_t = K_{t+1}$.

4.6 Financial markets

As stated earlier, households can save in two assets; risky equity and safe bonds (firm debt). There are borrowing constraints for both assets, so the lowest amounts of equity and debt that households can hold in period t are respectively: κ^s and κ^b. Markets are assumed to be incomplete, in the sense that there are no markets for the assets contingent on the realization of individual idiosyncratic shocks.

4.7 Government

The government has a twofold function in the model. First, in each period t, the government has to collect enough tax revenue to finance $G_t = \eta Y_t$, which is equal to a fraction η of the overall production of the economy in the period t. The government balances the budget by a labor tax τ_t^{lb}, and capital income taxes: τ_t^s, τ_t^b, which are the taxes on income from shares and bonds, respectively. An important simplifying assumption is that the government is forced to have a tax rate on bonds as a set difference between a tax rate on equity return and a constant C^f: $\tau_t^b = \tau_t^s - C^f$. In each period t, tax rates τ_{t+1}^s and τ_{t+1}^b have to be known. Therefore, to accommodate the variation in tax revenue collected by the capital tax rates (which depend on the realization of the aggregate shocks z_{t+1} and δ_{t+1}, the government balances the budget with a special tax on labor: τ_{t+1}^{lb}. A more realistic setting would have been to allow the government to run a budget deficit, but this is not computationally feasible since it would require the introduction of the additional state variable; the government debt (Gomes et al., 2013). However, running a balanced budget every period by adjusting a labor tax τ_t^{lb}, should not significantly influence the results since the labor supply in this model is exogenous (therefore, this tax is not distortive, but is redistributive).

The government budget constraint in period t is:

$$S_t R_t^s \tau_t^s + B_t R_t^b \tau_t^b + w_t L_t \tau_t^{lb} = G_t$$

or equivalently

$$\left((1-\lambda)K_t r_t^s + \lambda K_t r_t^b\right)\tau_t^s - \left(\lambda K_t r_t^b\right) C^f + w_t L_t \tau_t^{lb} = \eta Y_t$$

Concerning capital income taxes, the government follows a simple fiscal rule in period t, such that the expected revenue from capital income taxes in period $t+1$ is equal to the expected wasteful government expenditure:

$$E_t\left[\left((1-\lambda)K_{t+1}r^s_{t+1}+\lambda K_{t+1}r^b_{t+1}\right)\tau^s_{t+1}-\left(\lambda K_t r^b_t\right)C^f+w_{t+1}L_{t+1}\tau^{lb}_{t+1}\right]=E_t\left[\eta Y_{t+1}\right]$$

As a rule, after the realization of aggregate shocks in period $t+1$, revenues from capital income taxes will not be equal to ηY_{t+1}, and the difference will be collected (or returned to the taxpayers as a tax break) with the special labor income tax τ^{lb}. Therefore, contingent on the realization of the aggregate shocks in the period $t+1$, τ^{lb}_{t+1} is known in the period t.

Second, the government runs two social programs: social security (retirement benefits), and unemployment insurance, and are modeled as in Krueger et al. (2016). Both are financed by separate labor taxes. Social security is financed with a constant labor tax rate: τ^{lss}, and the revenues $T^{ss}_t=\frac{L_t}{\Pi_R}w_tL_t\tau^{lss}$ are equally distributed in period t to all retired households, irrespective of their past contributions. Unemployment benefits are financed with a labor tax rate τ^u_t. The amount of the unemployment benefits $g_{u,t}$ is determined by a constant ϕ, which represents the fraction of the average labor earnings that are paid to the unemployed agent. Therefore, $g_{u,t}=\phi w_tL_t$.

To satisfy the budget constraint the government has to tax labor with the tax rate:

$$\tau^u_t=\frac{1}{1+\frac{1-\Pi_u(z)}{\Pi_u(z)\phi}}$$

where Π_u is the share of the unemployed people in the total working-age population.

4.8 Household problem

Retired household i maximizes its lifetime utility subject to the following constraints:

$$c_{i,t}+s_{i,t+1}+b_{i,t+1}+\phi\mathbb{I}_{\{s_{i,t+1}\neq 0\}}\leq\omega_{i,t}$$

$$\omega_{i,t+1}=T_{ss,t+1}+\left[(1+r^s_{t+1}(1-\tau^s_{t+1}))s_{i,t+1}+(1+r^b_{t+1}(1-\tau^b_{t+1}))b_{i,t+1}\right]\frac{1}{v}$$

$$(c_{i,t},b_{i,t+1},s_{i,t+1})\geq\left(0,\kappa^b,\kappa^s\right)$$

Working age household i maximizes its expected lifetime utility subject to the constraints below:

$$c_{i,t} + s_{i,t+1} + b_{i,t+1} + \phi \mathbb{I}_{\{s_{i,t+1} \neq 0\}} \leq \omega_{i,t}$$

$$\omega_{i,t+1} = \begin{cases} w_{t+1} l_{i,t+1} (1 - \tau^l_{t+1}) + (1 + r^s_{t+1}(1 - \tau^s_{t+1})) s_{i,t+1} + (1 + r^b_{t+1}(1 - \tau^b_{t+1})) b_{i,t+1} & \text{if } e = 1 \\ g_{u,t+1} + (1 + r^s_{t+1}(1 - \tau^s_{t+1})) s_{i,t+1} + (1 + r^b_{t+1}(1 - \tau^b_{t+1})) b_{i,t+1} & \text{if } e = 0 \end{cases}$$

$$(c_{i,t}, b_{i,t+1}, s_{i,t+1}) \geq \left(0, \kappa^b, \kappa^s\right)$$

First order conditions imply that the following equations are satisfied:

$$1 \geq E_t \left\{ m^j_{i,t,t+1} (1 + r^s_{t+1}(1 - \tau^s_{t+1})) \right\} \;\; and \; s^j_{i,t+1} \geq \kappa^s$$

$$1 \geq E_t \left\{ m^j_{i,t,t+1} (1 + r^b_{t+1}(1 - \tau^b_{t+1})) \right\} \;\; and \; b^j_{i,t+1} \geq \kappa^b$$

where pricing kernel is $m^j_{i,t,t+1} = \beta \left[\frac{c^j_{i,t+1}}{c^j_{i,t}} \right]^{\frac{(1-\alpha)(\rho-1)}{\rho}} (R^a_{t+1})^{\frac{1-\alpha}{\rho}}$ for household i between periods t and $t+1$, where R^a_{t+1} is the gross post tax return on asset a, and j denotes if the household is of working age W, or retired R. Furthermore, define $q^s_{t,t+1}$ and $q^b_{t,t+1}$ as the stochastic marginal rates of substitution of households that are unconstrained at period t in their choices of shares and bonds, respectively (they have an interior solution to their portfolio choice problem).

Asset pricing equations are:

$$1 = E_t \left\{ q^s_{t,t+1} (1 + r^s_{t+1}(1 - \tau^s_{t+1})) \right\}$$

$$1 = E_t \left\{ q^b_{t,t+1} (1 + r^b_{t+1}(1 - \tau^b_{t+1})) \right\}$$

or more concisely:

$$1 = E_t \left\{ q^a_{t,t+1} R^a_{t+1} \right\}$$

4.9 Recursive household problem

It is possible to define the household's problem recursively: Retired households:

$$v_R(\omega; z, \mu, \delta) = \max_{c,b',s'} \left\{ c^{1-\rho} + v\beta E_{z',\mu',\delta'|z,\mu,\delta}[v_R(\omega'; z', \mu', \delta')^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

subject to:

$$c + s' + b' + \phi \mathbb{I}_{\{s' \neq 0\}} = \omega$$

$$\omega' = T'_{ss} + \left[s'(1 + r'^s(1 - \tau'^s)) + b'(1 + r'^b(1 - \tau'^b)) \right] \frac{1}{v}$$

Working age households:

$$v_W(\omega, e, l; z, \mu, \delta) =$$

$$\max_{c,b',s'} \left\{ c^{1-\rho} + \beta E_{e',l',z',\mu',\delta'|e,l,z,\mu,\delta}[(1-\theta) v_W(\omega', e', l'; z', \mu', \delta')^{1-\alpha} + \theta v_R(\omega', e', l'; z', \mu', \delta')^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

subject to:

$$c + s' + b' + \phi \mathbb{I}_{\{s' \neq 0\}} = \omega$$

$$\omega' = \begin{cases} w'l'(1 - \tau'^l) + s'(1 + r'^s(1 - \tau'^s)) + b'(1 + r'^b(1 - \tau'^b)) & \text{if } e = 1 \\ g'_u + s'(1 + r'^s(1 - \tau'^s)) + b'(1 + r'^b(1 - \tau'^b)) & \text{if } e = 0 \end{cases}$$

$$(c, b', s') \geq \left(0, \kappa^b, \kappa^s\right)$$

where ω is the vector of individual wealth of all agents, μ is the probability measure generated by set $\Omega x E x L$, $\mu' = \Gamma(\mu, z, z', d, d')$ is a transition function and $'$ denotes the next period.

4.10 General equilibrium

The economy-wide state is described by $(\omega, e; z, \mu, d)$. Therefore, the individual household policy functions are: $c^j = g^{c,j}(\omega, e, l; z, \mu, d)$, $b'^j = g^{b,j}(\omega, e, l; z, \mu, d)$ and $s'^j = g^{s,j}(\omega, e, l; z, \mu, d)$, and laws of motion for the aggregate capital is $K' = g^K(\omega, e, l; z, \mu, d)$.

A recursive competitive equilibrium is defined by the set of individual policy and value functions $\left\{v_R, g^{c,R}, g^{s,R}, g^{b,R}, v_W, g^{c,W}, g^{s,W}, g^{b,W}\right\}$, laws of motion for the aggregate capital g^K, a set of pricing functions $\left\{w, R^b, R^s\right\}$, government policies in period

t: $\{\tau^{lb}, \tau^{u}, \tau^{s}, \tau^{b}\}$ and tax rates contingent on the aggregate states in period $t+1$: $\{\tau'^{lb}, \tau'^{u}, \tau'^{s}, \tau'^{b}\}$, and a forecasting equation g^L, such that:

1. The laws of motion for the aggregate capital g^K and the aggregate "wage function" w, given the taxes satisfy the optimality conditions of the firm.

2. Given $\{w, R^b, R^s\}$, the laws of motion Γ, the exogenous transition matrices $\{\Pi_z, Pi_e, Pi_l\}$, the forecasting equation g^L, the laws of motion for the aggregate capital g^K, and the tax rates, the policy functions $\{g^{c,j}, g^{b,j}, g^{s,j}\}$ solve the household problem.

3. Labor, shares and the bond markets clear:

   - L= $\int eld\mu$
   - $\int g^{s,j}(\omega, e, l; z, \mu, \delta)\, d\mu = (1-\lambda)K'$
   - $\int g^{b,j}(\omega, e, l; z, \mu, \delta)\, d\mu = \lambda K'$

4. The laws of motion $\Gamma(\mu, z, z', \delta, \delta')$ for μ is generated by the optimal policy functions $\{g^c, g^b, g^s\}$, the laws of motion for aggregate capital g^K and by the transition matrices for the shocks. Additionally, the forecasting equation for aggregate labor is consistent with the labor market clearing: $g^L(z', \delta') = \int \epsilon ld\mu$

5. Government budget constraints are satisfied:

$$E_t\left[\left((1-\lambda)K_{t+1}r^s_{t+1} + \lambda K_{t+1}r^b_{t+1}\right)\tau^s_{t+1} - \left(\lambda K_t r^b_t\right)C^f + w_{t+1}L_{t+1}\tau^{lb}_{t+1}\right] = E_t\left[\eta Y_{t+1}\right]$$

$$g_{u,t} = \phi w_t L_t$$

$$T^{ss}_t = \frac{L_t}{\Pi_R} w_t L_t \tau^{lss}$$

$$\tau^u_t = \frac{1}{1 + \frac{1-\Pi_u(z)}{\Pi_u(z)\phi}}$$

4.11 Parametrization and calibration

The model is calibrated to a quarterly frequency. The goal of the calibration is to match the important patterns of wealth inequality and portfolio choice in the US economy.

Moreover, it is calibrated to match the capital-output ratio excluding housing, following Algan et al. (2009). The capital does not include housing, even though housing is a substantial part of assets owned by poor households (Kuhn and Rios-Rull, 2016). Housing is not included for the reason of simplicity and the fact that not all of the value of the housing should be considered as savings, because housing has a use-value of providing accommodation and can be partly considered consumption. However, excluding (mostly risky, since it is financed by mortgages) housing wealth, can cause the financial capital portfolio allocation to be misspecified because it implies excessively low background risk. Parameter η, which governs how much asset tax revenue the government has to collect in each period as the share of the output. The value is set as an approximation of how much the US federal government collected in 2016 from taxes from savings, as a percentage of GDP.[7] Overall, there are four possible aggregate states of the economy, since both TFP and capital depreciation shocks can take two possible values.

Preferences, firm and households constraints

Table 1: Internally-calibrated parameters

Parameter	Symbol	Value	Target
Discount factor	β	0.877	Capital-Output ratio: 7
Subsistence constraint	γ	0.036	Portfolio choice pattern
Quarterly stock market participation costs	ϕ	0.002	Share of households with no equity 46%

Table 2: Externally-calibrated parameters

Parameter	Symbol	Value	Target
Intertemporal elasticity of substitution	$\frac{1}{\rho}$	0.5	Capital-Output ratio: 7
Expected depreciation rate	$E(\delta)$	0.033	Equity premium $1-2\%$
Chance of not retiring	θ	.994	Average working duration 40 years
Chance of not dying	v	0.983	Average retirement duration 15 years
Tax advantage of debt	τ^s	0.3	Hennessy and Whited (2005)
Capital share	Δ	0.4	Algan et al. (2009)

[7] The revenues taken into account are from taxes of corporate income and gains, and one third (capital share) of personal income, profits, and gains.

Table 3: Parameters to generate sizable equity premium

Parameter	Symbol	Value
Risk aversion	α	10
Variance of depreciation rate	$\sigma^2(\delta)$	0.0001

Table 4: Other parameters

Parameter	Symbol	Value
Social security tax	τ^{lss}	0.06
Unemployment replacement rate	η	0.042
Borrowing constraint: bonds	κ^b	0.00
Borrowing constraint: stocks	κ^s	0.0

Idiosyncratic labor income shocks

For the idiosyncratic labor income shocks transition matrix, I use the same values as Pijoan-Mas (2007) and Algan et al. (2009).

$$\Pi_l = \begin{bmatrix} 0.9850 & 0.0100 & 0.0050 \\ 0.0025 & 0.9850 & 0.0125 \\ 0.0050 & 0.0100 & 0.9850 \end{bmatrix}$$

For the individual labor productivity levels, the following values are used: $l \in \{44, 8, 1.5\}$ (they differ slightly from the ones used by Algan et al. (2009)). This type of modeling the labor productivity process allows the generation of a realistic size of earnings and wealth inequality, while keeping the possible number of states relatively low.

The average unemployment duration during booms is set to 1.6 quarters, while for the recession, it is set to 2.8 quarters.

Table 5: Quarterly statistics

	K/Y	Wealth GINI	R^b	$E(R^s_{pt}) - R^b_{pt}$
Data	7.01	0.78	0.23	1.0 - 2.0
Economy	7.03	0.59	1.83	0.09

The data figures in Table 5. are taken from Algan et al. (2009). The reported returns from the model are post-tax.

Table 6: Wealth distribution: Owned share of overall wealth %

Quintile	Data	Model
Q1	-0.2	1.4
Q2	1.2	3.67
Q3	4.6	11.6
Q4	11.9	21.0
Q5	82.5	62.4
T1 %	33.5	4.7

The data in Table 5. is taken from Krueger et al. (2016), who use the Panel Study of Income Dynamics (PSID). The model replicates the bottom tail of the wealth distribution fairly closely, but it does not generate a thick enough top tail of the distribution, which is common in similar types of models (without discount rate heterogeneity and without shocks to the asset returns).

Figure 1)

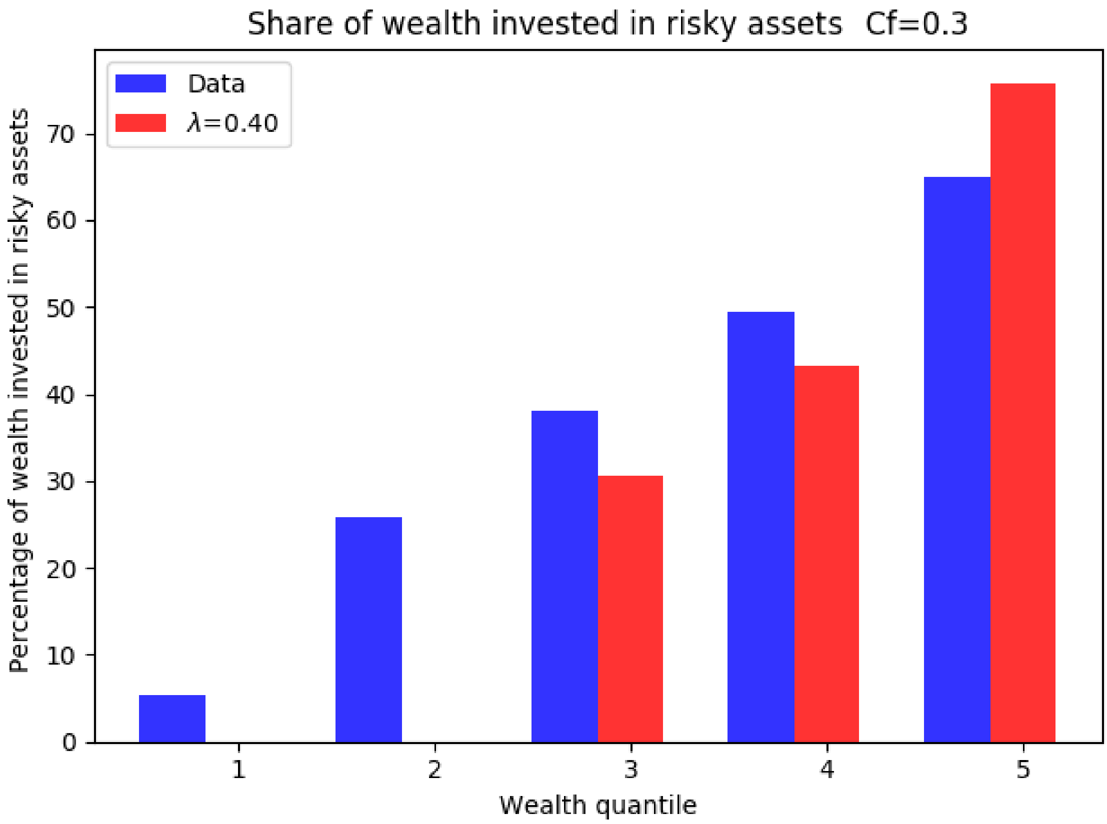

Figure 1 reports the data from Chang et al. (2018), who use SCF. The model matches the portfolio choices along the wealth distribution reasonably well. The only major deviations seem to be the not risky enough portfolio in the second quintile of the distribution, and excessively risky portfolio of the richest quintile.

4.12 Solution method

It is well known that solving these types of models is difficult since the state variables include the cross-sectional distribution of agents over the wealth for each (un)employment status. When the model features aggregate risk, the cross-sectional distribution of agents over the wealth is a time-varying infinite-dimensional object. In this model, I follow the approach of Krusell and Smith (1997, 1998), who reduce the state-space to include only a finite set of cross-sectional distribution moments. In the simulation of the model, I use a non-stochastic simulation routine described by Young (2010) to keep track of the household wealth distribution. The procedure is the alternative to the Monte Carlo simulation procedure, and is used to speed up the convergence of wealth distribution while using the fine grid for household wealth.

The approximate equilibrium laws of motion for capital and bond interest rate are the following:

$$lnK' = a_0(z,d) + a_1(z,d)lnK$$

and

$$lnP^e = b_0(z,d) + b_1(z,d)lnK'$$

While solving the individual household problem, given the aggregate laws of motion, I use the "endogenous grid method" proposed by Carroll (2006), augmented to allow for two choice variables by the agents. This method reduces the computational time, as it avoids the root-finding process. In the benchmark model, the R^2 of the laws of motion are 99.8% for capital and 99.9% for equity premium.

The numerical implementation of the solution algorithm is discussed in more detail in Appendix D.

5 Exercises and results

The main computational exercises are in changing the ratio of the taxes of safe (bonds) and risky (stocks) assets $C^f = \tau_s - \tau_b$. C^f can be thought of as a measure for the debt tax shield. This exercise keeps the government revenue from the capital taxes the same, but it changes the composition of the raised revenues (between revenues from stocks and bonds). The benchmark value for C^f is set at $C^f = 0.3$, which is taken from Hennessy and Whited (2005). The welfare is measured from the perspective of the utilitarian social planner.

5.1 Exercise 1: Eliminating the tax wedge

In the first exercise, the tax wedge (or debt tax shield) is eliminated, which means that the C^f is set to 0. All other parameters are held exactly the same.

Table 7: Results: Eliminating the tax wedge

Unconditional moments	$C^f = 0.0$ Economy	$C^f = 0.3$ Economy
Post-tax E$(r^{sP} - r^{bP})$	0.448%	0.401%
Pretax E(r^b)	1.93%	1.55%
Pretax E(r^s)	2.53%	2.80%
Post-tax E(r^{bP})	1.55%	1.59%
Post-tax E(r^{sP})	2.00%	1.99%
Wealth GINI	0.5833	0.5831
$E(K)$	5.766	5.747
Share of wealth by quintile (%)		
1st	1.85%	1.80%
2st	3.19%	3.26%
3st	11.40%	11.46%
4st	20.91%	20.84%
5st	62.64%	62.62%

Table 8: $C_f = 0$: Welfare gains from abolishing the beneficial tax treatment of debt

	Wealth change				
Quintile	1	2	3	4	5
% change	-0.71	+0.00	-0.18	-0.25	-0.32

The elimination of the debt tax shield is found to be welfare reducing and is equivalent to a 0.3% permanent decrease in consumption (including the transition path of the economy). The decline in welfare is mainly due to the decreased return in the safe asset, in which the poor households (households with a high marginal utility of wealth) mostly save. On the other hand, the households that gain are the wealthy households (which have a lower marginal utility of wealth), as they save mostly in the risky equity. Therefore, the policy that taxes the risky asset more effectively functions as insurance. It also serves as insurance for the equity owners, since it reduces the variance of the after-tax returns. This is useful for households, as the income from labor and asset returns are correlated. Therefore, the households that would benefit the most from this are households that have significant income from both labor and assets. Furthermore, wealth inequality increases slightly, as the poorer households save less in the economy without the debt tax shield. The welfare gains by quintile can be seen in Table 8.

5.2 Exercise 2: Finding the optimal tax wedge

In this section, the optimal C^f for the long-run is calculated. The optimal level of debt tax shield is found to be $C^f = 0.60$. This means that in the optimum, the returns on the equity are heavily taxed, and the returns to bonds are heavily subsidized. As discussed earlier, this helps all, and especially poor households, to insure themselves against the bad aggregate shocks. The reason subsidizing a return to safe assets is particularly useful for poor households, is that it enables them to receive high returns on savings without exposing themselves to the uncertainty of equity returns and without paying equity market participation costs. The share of households participating in the stock markets drops slightly from 41.75% to 40.23%, when the economy moves from $C^f = 0.30$ to $C^f = 0.60$. The reported welfare changes are in terms on % of consumption equivalent variation. The aggregate capital is decreasing when the tax wedge C^f increases. This is partly due to the fact that the savings in a risky asset are more elastic to the change in the net return.[8].

[8] The reasons for this are discussed in the two-period model section.

Table 9: Results: Optimal long-run C^f

Moments	$C^f = 0.0$	$C^f = 0.3$	$C^f = 0.60$	$C^f = 0.90$
Welfare change from the benchmark model: $C_f = 0.3$ [9]	-0.028%	0.0%	$+0.105\%$	-0.001%
Wealth GINI	0.5833	0.5831	0.5826	0.5829
$E(K)$	5.766	5.749	5.740	5.726
Pretax E(r^b)	1.93	1.55	1.26	1.06
Pretax E(r^s)	2.53	2.80	2.99	3.15
Post-tax E(r^{bP})	1.55	1.59	1.61	1.63
Post-tax E(r^{sP})	2.00	1.99	1.98	1.98
Post-tax E($r^{sP} - r^{bP}$)	0.448	0.401	0.371	0.354

5.3 Exercise 3: Changing leverage with the changing tax wedge

An important caveat of the performed analysis is that the firm leverage was taken as exogenous. However, it is intuitive to expect that the firms would adjust their leverage after a change in taxes to optimize their financing policy in an attempt to avoid excessive taxation. To control for the possible change in firm financing policy, this exercise performs the change in the tax wedge, exactly as in Exercise 1, but at the same time, exogenously changes the leverage.

The effect of changing the leverage does not overturn or dampen the effects of simulated tax reforms on welfare and wealth inequality. On the contrary, the effect of eliminating the debt tax shield on welfare and inequality is underestimated. As shown in Table 10., when the change of leverage is taken into consideration, the welfare and inequality changes are even larger because the welfare gains from increasing the debt tax shield in the model mostly stem from the increased after-tax return on the safe asset (in which poor and constrained agents save). Therefore, when the debt tax shield is decreased, the firms would presumably decrease their debt financing. This decreases the issuance of bonds and consequently decreases their return.

Table 10: Results: Economies with exogeneously different leverages

Moments	$C^f = 0.0, \lambda = 0.40$	$C^f = 0.0, \lambda = 0.20$	$C^f = 0.6, \lambda = 0.40$	$C^f = 0.6, \lambda = 0.60$
Welfare change	-0.028%	-0.080%	$+0.105\%$	$+0.141\%$
Wealth GINI	0.5833	0.5838	0.5826	0.5819
$E(K)$	5.766	5.763	5.740	5.741
$\mathrm{E}(r^{sP} - r^{bP})$	0.448	0.343	0.371	0.478

6 Conclusion

This paper considers the macroeconomic consequences of differential taxation of risky and safe financial assets. The social planner may wish to tax safe assets at the lower rate in order to reduce the tax burden of poor households, that mainly save is safe assets. The theoretical part of the paper finds that the optimal difference between taxes on risky and safe assets is increasing in wealth inequality. The quantitative part of the paper finds that 1) the elimination of the debt tax shield is welfare reducing, and it is equivalent to a permanent consumption decrease of 0.3%, and 2) that the optimal tax shield is larger than in the current US tax code. In the general equilibrium model, the distortionary taxation that taxes the risky asset more is useful for the utilitarian social planner for multiple reasons: it shifts the tax burden from poor households owning safe assets to wealthy households holding equity, and it reduces the variance of the after-tax returns of the risky asset, which is beneficial for equity owners across the board.

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7 Appendix A. Two-period Ramsey model and proofs

A.1. Proof of Lemma 1. The proof closely follows Diamond (1975).

Government maximizes:

$$\max_{\tau_r \tau_s} \chi(p, m)$$

s.t.

$$\tau_r(\sum_{h=1}^{2} x_r^h(p, m^h)) + \tau_s(\sum_{h=1}^{2} x_s^h(p, m^h)) \geq G$$

The FOCs are the following, for the two tax rates for goods k:

$$\sum_{h=1}^{H} \frac{\partial \chi}{\partial V^h} \frac{\partial V^h}{\partial \tau_k} = -\lambda p_k \sum_{h=1}^{H} \left\{ x_k^h(p, m^h) + \sum_{j=1}^{J} \tau_j \frac{\partial x_j^h(p, m^h)}{\partial p_k} \right\}$$

Using the Roy's identity we get:

$$\frac{\partial V^h}{\partial q_k} = -\frac{\partial V^h}{\partial m} x_k^h$$

$$\frac{\partial V^h}{\partial q_k} = -\alpha^h x_k^h$$

And α is defined as agent $h's$ marginal utility of income.

From the Slutsky equation:

$$\frac{\partial x_j^h}{\partial p_k} = \frac{\partial \hat{x}_j^h}{\partial p_k} - x_k^h(p, m^h) \frac{\partial x_j^h}{\partial m}$$

Using the Slutsky equation and Roy's identity, we get:

$$\sum_{h=1}^{H} \frac{\chi}{\partial V^h} \alpha^h x_k^h = \lambda p_k \sum_{h=1}^{H} \left\{ x_k^h(p, m^h) + \sum_{j=1}^{J} \tau_j p_j \left[\frac{\partial \hat{x}_j^h}{\partial p_k} - x_j^h(p, m^h) \frac{x_j^h(p, m^h)}{\partial m} \right] \right\}$$

rewriting:

$$\sum_{j=1}^{J} \tau_j p_j \left(\sum_{h=1}^{H} \frac{\partial x_j^h}{\partial p_k} \right) = \frac{\sum_{h=1}^{H} \frac{\partial \chi}{\partial V^h} \alpha^h x_k^h}{\lambda} - \sum_{h=1}^{H} x_k^h(p, m^h) + \sum_{h=1}^{H} x_k^h(p, m^h) \sum_{j=1}^{J} \tau_j \frac{\partial x_j^h(p, m^h)}{\partial m}$$

Using the symmetry of the substitution matrix

$$\frac{\partial \hat{x}_j^h}{\partial p_k} = \frac{\partial \hat{x}_k^h}{\partial p_j}$$

the following is obtained:

$$\sum_{j=1}^{J} p_j\tau_j\Big(\sum_{h=1}^{H}\frac{\partial x_j^h}{\partial p_k}\Big) = \frac{\sum_{h=1}^{H}\frac{\partial\chi}{\partial V^h}\alpha^h x_k^h}{\lambda} - \sum_{h=1}^{H} x_k^h(p,m^h) + \sum_{h=1}^{H} x_k^h(p,m^h)\sum_{j=1}^{J}\tau_j\frac{\partial x_j^h(p,m^h)}{\partial m}$$

To simplify the expression, the following notation is used:

$$X_k = \sum_{h=1}^{H} x_k^h(p,m)$$

where X_k is the total demand for good k

$$\beta^h = \frac{\frac{\partial\chi}{\partial V^h}\alpha^h}{\lambda} + \sum_{j=1}^{J}\tau_j\frac{\partial x_j^h(p,m^h)}{\partial m}$$

where β^h is the so called "net social marginal utility of income for agent h"

$$\bar{\beta} = \sum_{h=1}^{H}\frac{\beta^h}{H}$$

Now, it is possible to rewrite the formula:

$$\sum_{j=1}^{J} p_j\tau_j\Big(\sum_{h=1}^{H}\frac{\partial\hat{x}_k^h}{\partial p_j}\Big) = -X_k\Big(1-\sum_{h=1}^{H}\beta^h\frac{x_k^h}{X_k}\Big)$$

Now, denote by θ_k the empirical covariance between β^h and $h's$ consumption of good k:

$$\theta_k = cov\Big(\frac{\beta^h}{\bar{\beta}},\frac{x_k^h}{\bar{X}_k}\Big) = \frac{1}{H}\sum_{h=1}^{H}\Big(\frac{\beta^h}{\bar{\beta}}-1\Big)\Big(\frac{x_k^h}{\bar{X}_k}-1\Big)$$

Positive θ_k indicates that good k is mostly consumed by agents with higher β^h (in the context of the model; poor agents).

θ_k can also be expressed as:

$$\theta_k = \sum_{h=1}^{H}\frac{\beta^h x_k^h}{X_k} - 1$$

Finally, the standard "many-person" Ramsey optimal rule formula can be written:

$$-\frac{\sum_{j=1}^{J}\tau_j p_j(\sum_{h=1}^{H}\frac{\partial\hat{x}_k^h}{\partial p_j})}{X_k} = 1-\bar{\beta}-\bar{\beta}\theta_k \tag{2}$$

■

A.2. Proof of Proposition 1.

$$\frac{U_R(kR,kS)}{U_S(kR,kS)} = \frac{U_R(R,S)}{U_S(R,S)}$$

It is a well-established result in the Ramsey framework that in the case of homogeneous utility there should be uniform taxation of goods: risky and safe assets should be taxed at the same rate.

Marshalian demand for the risky asset is:

$$x_r^h = \frac{\frac{m^h}{p_s}(A-1)}{r_g - r_b A + \frac{p_r}{p_s}(A-1)}$$

where

$$A = \left(\frac{r_g - \frac{p_r}{p_s}}{\frac{p_r}{p_s} - r_b} \right)^{\frac{1}{\alpha}} > 1$$

To see this, first consider a case in which there is no initial inequality: $m^h = \frac{\sum_h^H m^h}{H}$

$$\frac{\tau_r}{1+\tau_r}\hat{\epsilon}_{r,r} + \frac{\tau_s}{1+\tau_s}\hat{\epsilon}_{r,s} = 1 - \bar{\beta} - \bar{\beta}\theta_k$$

rearranging:

$$\frac{\frac{\tau_s}{1+\tau_s}}{\frac{\tau_r}{1+\tau_r}} = \frac{-\hat{\epsilon}_{r,r} + \hat{\epsilon}_{s,r}}{-\hat{\epsilon}_{s,s} + \hat{\epsilon}_{r,s}}$$

Rewriting in terms of uncompensated (Marshalian) elasticicities, using the Slutsky equation:

$$\frac{-(\epsilon_{r,r} + \epsilon_{r,m}B_r^h) + \epsilon_{s,r} + \epsilon_{s,m}B_r^h}{-(\epsilon_{s,s} + \epsilon_{s,m}B_s^h) + \epsilon_{r,s} + \epsilon_{r,m}B_s^h}$$

Rewriting using the "symmetry" equation ($\epsilon_{i,j} = \epsilon_{j,i}\frac{B_j}{B_i} + B_j(\epsilon_{j,m} - \epsilon_{i,m})$), where B_i is a budget share of a good i):

$$\frac{-(\epsilon_{r,r} + \epsilon_{r,m}B_r^h) + \epsilon_{r,s}\frac{B_r^h}{B_s^h} + \epsilon_{r,m}B_r^h}{-(\epsilon_{s,s} + \epsilon_{s,m}B_s^h) + \epsilon_{s,r}\frac{B_s^h}{B_r^h} + \epsilon_{s,m}B_r^h} = \frac{-\epsilon_{r,r} + \epsilon_{r,s}\frac{B_r^h}{B_s^h}}{-\epsilon_{s,s} + \epsilon_{s,r}\frac{B_s^h}{B_r^h}}$$

Using the homogeneity condition ($\epsilon_{r,s} + \epsilon_{r,r} + \epsilon_{r,m} = 0$):

$$\frac{\epsilon_{r,s} + \epsilon_{r,m} + \epsilon_{r,s}\frac{B_r^h}{B_s^h}}{\epsilon_{s,r} + \epsilon_{s,m} + \epsilon_{s,r}\frac{B_s^h}{B_r^h}} = \frac{\epsilon_{r,m} + \epsilon_{r,s}\frac{B_s^h + B_r^h}{B_s^h}}{\epsilon_{s,m} + \epsilon_{s,r}\frac{B_s^h + B_s^h}{B_r^h}}$$

Using the "symmetry" property again:

$$\frac{\frac{\tau_s}{1+\tau_s}}{\frac{\tau_r}{1+\tau_r}} = \frac{\frac{B_s^h + B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{r,m} + (B_s^h + B_r^h)(\epsilon_{s,m} - \epsilon_{r,m})}{\frac{B_s^h + B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{s,m}} \tag{3}$$

If there is no labor (leisure), budget shares sum up to 1 ($B_s^h + B_r^h$), and there should be no tax distortion.

Marshalian demand for the risky asset is:

$$x_r^h = \frac{\frac{m^h}{p_s}(A-1)}{r_g - r_b A + \frac{p_r}{p_s}(A-1)}$$

where

$$A = \left(\frac{r_g - \frac{p_r}{p_s}}{\frac{p_r}{p_s} - r_b}\right)^{\frac{1}{\alpha}} > 1$$

Marshalian demand for the safe asset is:

$$x_s^h = \frac{\frac{m^h}{p_r}(Ar_b - r_g)}{1 - A + \frac{p_s}{p_r}(Ar_b - r_g)}$$

Both are linear in m^h.

Computing the income elasticities:

$$\epsilon_{r,m} = \frac{m^h}{x_r^h}\frac{dx_r^h}{dm^h} = 1 = \epsilon_{s,m}$$

Therefore, from (3), we see that there will be uniform taxation between two assets.

The result extends to the case with initial inequality, as the elasticities do not depend on m^h:

$$\frac{\partial^3 x_j^h}{\partial p_i \partial^2 m^h} = 0$$

■

A.3. Proof of Proposition 2. Consider the equation (2):

$$\frac{\frac{\tau_s}{1+\tau_s}}{\frac{\tau_r}{1+\tau_r}} = \frac{\frac{B_s^h+B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{r,m} + \left(B_s^h + B_r^h\right)(\epsilon_{s,m} - \epsilon_{r,m})}{\frac{B_s^h+B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{s,m}}$$

Adding and subtracting $\epsilon_{s,m}$ in the numerator;

$$\frac{\frac{\tau_s}{1+\tau_s}}{\frac{\tau_r}{1+\tau_r}} = \frac{\frac{B_s^h+B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{s,m} + \left(1 - B_s^h + B_r^h\right)(\epsilon_{r,m} - \epsilon_{s,m})}{\frac{B_s^h+B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{s,m}}$$

Because $B_s^h + B_r^h = 1$, we have:

$$\frac{\frac{\tau_s}{1+\tau_s}}{\frac{\tau_r}{1+\tau_r}} = 1$$

■

However, if there are more than the two mentioned assets $\left(B_s^h + B_r^h\right) \leq 1$, and if $\epsilon_{r,m} > \epsilon_{s,m}$, the safe asset should be taxed more.

$$\epsilon_{r,m} = \frac{m^h}{x_r^h}\frac{dx_r^h}{dm^h} = \frac{\frac{m^h}{p_s}(A-1)}{\frac{m^h}{p_s}(A-1) - L_g} > 1$$

$$\epsilon_{s,m} = \frac{m^h}{x_s^h}\frac{dx_s^h}{dm^h} = \frac{\frac{m^h}{p_s}(r_g - r_b A + \frac{p_r}{p_s}(A-1)) - p_r(A-1)}{\frac{m^h}{p_s}(r_g - r_b A + \frac{p_r}{p_s}(A-1)) - p_r(A-1) + L_g} < 1$$

Therefore, $\epsilon_{r,m} > \epsilon_{s,m}$, and the safe asset would be taxed at the higher rate in the optimum.

A.4. Proof of Proposition 3. Formally, in examining equation 1: θ_s is positive, and θ_r is negative. For example, examine θ_r:

$$\theta_r = \sum_{h=1}^{H} \frac{\beta^h x_k^h}{\bar{\beta} X_k} - 1$$

Let us consider a case with only two agents: rich and poor, and denote the rich agent with superscript C, and the poor agent with superscript P. They differ in the initial income $m^C > m^P$.

$\beta^P > \beta^C$ because $\alpha^P > \alpha^C$, and the term $\frac{\partial x_j^h(p,m^h)}{\partial m}$ cannot compensate for this, because it would imply that the tax policy is not optimal. Therefore, the net marginal utility of one additional unit of income is greater for the poor agent. It immediately follows that $\frac{\beta^P}{\beta^C}$ is increasing in $\frac{m^C}{m^P}$.

Furthermore, $\frac{x_r^P}{x_r^P + x_r^C} < 0.5$, because, as we have seen before $\epsilon_{r,m} > 1$.

Using these facts:

$$\theta_r = \frac{\overbrace{\beta^P}^{>\bar{\beta}} \overbrace{x_r^P}^{<x_r^C} + \overbrace{\beta^C}^{<\bar{\beta}} \overbrace{x_r^C}^{>x_r^P}}{\bar{\beta}(x_r^P + x_r^C)} - 1$$

Since the average of β^C and β^P is $\bar{\beta}$, this implies that $\theta_r < 0$. Since there are only two assets, and θ_r is negative, θ_r must be positive: $\theta_s > 0$.

Examining the optimal many-person Ramsey taxation formulas, this implies that the risky asset should be taxed more (compared to the safe asset), the larger the initial inequality (ceteris paribus) is. In other words, the mean-preserving spread in M results in a higher taxation of the risky asset.

The extension to the case with H agents is straightforward. Namely, the LHS of (3) is not changed by the mean-preserving spread in m, as the elasticities and Marshallian demands are linear in income m^h. Therefore, LHS of (3) is just a function of m ($m = \sum_h^H m^h$). Again, the proof is obvious when the following property is used:

$$\frac{\partial^3 x_j^h}{\partial p_i \partial^2 m^h} = 0$$

■

Appendix A5. Introduction of Non-taxable good (consumption, leisure)

Consider the case where now the agents do not have only fixed wealth m^h, but can also increase their investment budget by decreasing l, which can be interpreted as either leisure or consumption in the first period.

$$U(R^h, S^h) = \frac{1}{2}\Big\{\frac{(R^h r_g + S^h + L_g)^{1-\alpha}}{1-\alpha} + \frac{(R^h r_b + S^h + L_b)^{1-\alpha}}{1-\alpha}\Big\} + v(l^h)$$

s.t.

$$R^h p_r + S^h p_s + l^h = m^h$$

v is well behaved and increasing in l.

Here l is a non-taxed good, in the tradition of Ramsey literature. Define the wealth that the agent spends on investing as : $W^h = m^h - l^h$. Furthermore, for simplicity, assume that there is no inequality and $m^h = 0$ $\forall h$. The optimal taxation equation looks like this:

$$\frac{\frac{\tau_s}{1+\tau_s}}{\frac{\tau_r}{1+\tau_r}} = \frac{\frac{B_s^h + B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{s,W} + \left(1 - B_s^h + B_r^h\right)(\epsilon_{r,W} - \epsilon_{s,W})}{\frac{B_s^h + B_r^h}{B_r^h}\epsilon_{s,r} + \epsilon_{s,W}}$$

However, now there are more than two budget elements, so $\left(B_s^h + B_r^h\right) \leq 1$, and $\epsilon_{r,W} > \epsilon_{s,W}$, the safe asset should be taxed more.

$$\epsilon_{r,W} = \frac{W^h}{x_r^h}\frac{dx_r^h}{dW^h} = \frac{\frac{W^h}{p_s}(A-1)}{\frac{W^h}{p_s}(A-1) - L_g} > 1$$

$$\epsilon_{s,W} = \frac{W^h}{x_s^h}\frac{dx_s^h}{dW^h} = \frac{\frac{W^h}{p_s}(r_g - r_b A + \frac{p_r}{p_s}(A-1)) - p_r(A-1)}{\frac{W^h}{p_s}(r_g - r_b A + \frac{p_r}{p_s}(A-1)) - p_r(A-1) + L_g} < 1$$

Therefore, $\epsilon_{r,W} > \epsilon_{s,W}$, and the safe asset would be taxed at the higher rate in the absence of inequality. Still, increase in inequality increases the optimal ratio of tax on the risky and tax on the safe asset. The difference with the case without l is that now, the complete taxable wealth $W^h = m^h - l^h$ is not fixed, since agents decide on l. Therefore, the standard result reemerges: assets with higher income elasticity should be taxed less.

8 Appendix B. Taxation of asset returns

In Appendix B, the case in which government taxes the returns on assets is considered.

The utilitarian social planner maximizes social welfare by choosing the taxes in the first period to collect the expected revenue $G = E\left[\sum_h^H R^h r_r t_r + \sum_h^H r_f S^h t_f\right]$. Where r_r has two equally likely possible realizations: r_g in a good, and r_b in a bad state. Denote the tax rate on the risky asset as t_r, and the tax rate on safe asset as t_f. Furthermore, denote the net returns: $R_g = r_g(1-t_r)$, $R_b = r_b(1-t_r)$ and $R_f = r_f(1-t_f)$.

Then, the problem of the households is:

$$\max_{R^h} \quad \frac{1}{2}u\left(Y^h R_f + R^h(R_g - R_f) + L_g\right) + \frac{1}{2}u\left(Y^h R_f + R^h(R_b - R_f) + L_b\right)$$

Taking the first order conditions and rearranging, we get the optimal choices of the amount of wealth invested in the risky asset:

$$R^{h*} = \frac{Y^h R_f(1-A) + L_b - L_g A}{AR_g - AR_f - R_b + R_f}$$

where $A = (\frac{R_b - R_f}{R_f - R_g})^{-\frac{1}{\alpha}}$.

This appendix verifies numerically, in a two-agent economy, that, even when the asset returns are taxed, the ratio of optimal taxes $\frac{t_r}{t_f}$ is increasing in the initial inequality $\frac{Y^C}{Y^P}$.

Consider the following example:

$$u(x) = \frac{x^\alpha}{1-\alpha}$$

with the following parameter values:

Table 11: Parameter values

r_g	r_b	r_f	α	G	$\sum_h^2 Y^h$	L_g	L_b
0.076	0.01	0.02	2	0.15	25	0.20	0.04

Figure 6. Taxation of asset returns

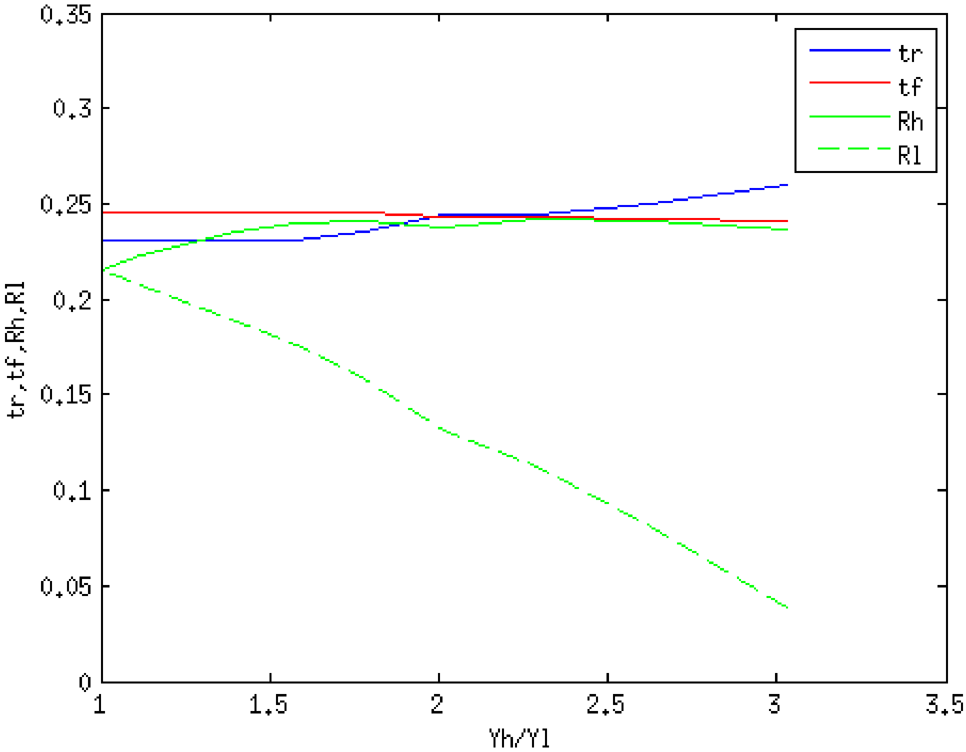
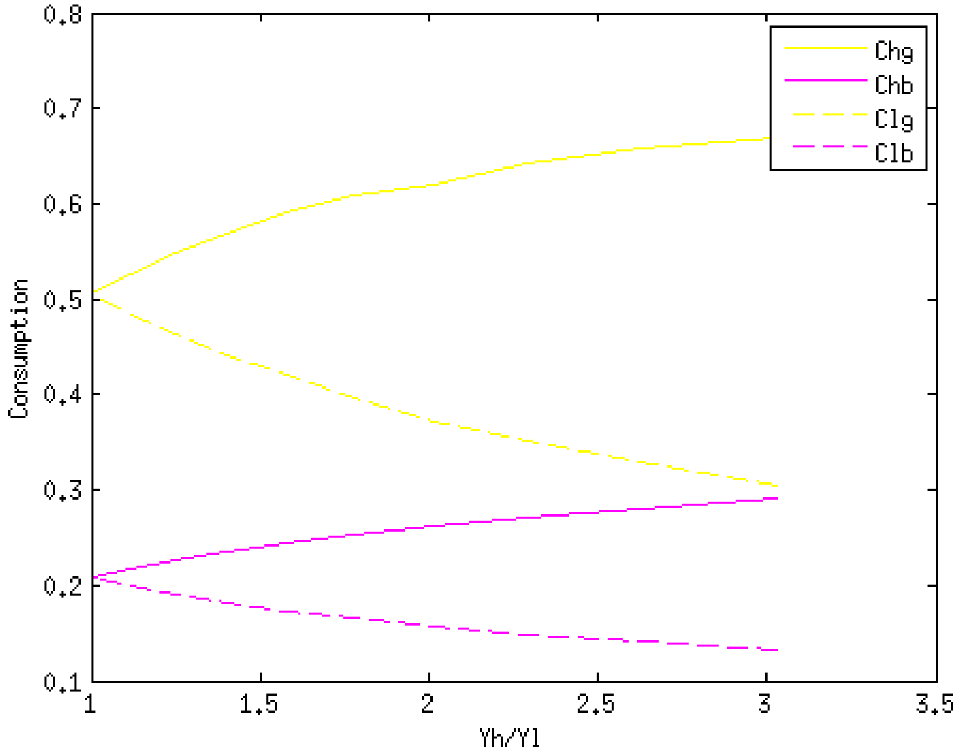


Figure 7. Taxation of asset returns: consumption in the second period



9 Appendix C. Solution algorithm

This appendix briefly describes the solution algorithm used to obtain the solution of the quantitative model. The algorithm uses the method used by Krusell and Smith (1997, 1998), which replaces the infinite-dimensional wealth distribution with a finite set of moments of wealth distribution, as a state variable.

Furthermore, for solving the individual problem, given the laws of motion for aggregate variables, the "endogenous grid method" proposed by Carroll (2006) is used. The method is augmented to allow for two choice variables (amount of savings and the composition of savings between stocks and bonds). I use a FORTRAN programming language for the numeric computation, since the computation is intensive, and requires a "low-level" programming language for the runtime of the program to be feasibly short.

1. Guess the law of motion for aggregate capital K_{t+1} and interest rate P_t^e. There will be for equations, as there are 4 possible aggregate state realizations (two realizations for TFP and two for depreciation shock).

2. Given the perceived laws of motion, solve the individual problem described in section 2.4.8. In this step endogenous grid method Carroll (2006) is used. Instead of constructing the grid on the state variable ω, and and searching for the optimal decision for savings $\tilde{\omega}$, this method creates a grid on the optimal savings amounts $\tilde{\omega}$, and evaluates the individual optimality conditions to obtain the level of wealth

ω at which it is optimal to save $\tilde{\omega}$. This way, the root-finding process is avoided, since finding optimal ω, given $\tilde{\omega}$, involves only evaluation of a function (households optimality condition). However, root finding process is necessary to find the optimal portfolio choice of the household, which is performed after finding the optimal pairs ω and $\tilde{\omega}$.

3. Simulate the economy, given the perceived aggregate laws of motion. To keep track of wealth distribution, instead of a Monte Carlo simulation, the method proposed by Young (2010) is used. For each realized value of ω, the method distributes the mass of agents between two grid points: ω_i and ω_{i+1}, where $\omega_i < \omega < \omega_{i+1}$, based on the distance of ω, based on Euclidean distance between ω_i, ω and ω_{i+1}. Do this in the following steps:

   (a) Set up an initial distribution in period 1: μ over a simulation grid $i = 1, 2, ...N_{sgrid}$, for each pair of efficiency and employment status, where N_{sgrid} is the number of wealth grid points. Set up an initial value for aggregate states z and d.

   (b) Find the bond interest rate in the given period R^b, which clears the market for bonds. This is performed by iterating on P^e, until the following equation is satisfied (bond market clears):
$$\sum_j g^{b,j}(\omega, e, l; z, d, K, P^e)d\mu = \lambda \sum_j \{g^{b,j}(\omega, e, l; z, d, K, P^e)d\mu + g^{s,j}(\omega, e, l; z, d, K, P^e)d\mu\}$$
   where $g^{b,j}(\omega, e, l; z, d, K, P^e)$ and $g^{s,j}(\omega, e, l; z, d, K, P^e)$ are the policy functions for bonds and shares, where j denotes the age of the household (working age or retired), that solve the following recursive household maximization problems: Retired households:
$$v_R(\omega; z, K, \delta, P^e) = \max_{c,b',s'} \left\{ c^{1-\rho} + v\beta E_{z',K',\delta',P^{e'}|z,K,\delta,P^e}[v_R(\omega'; z', K', \delta', P^{e'})^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$
   Working age households:
$$v_W(\omega, e, l; z, K, \delta, P^e) =$$
$$\max_{c,b',s'} \left\{ c^{1-\rho} + \beta E_{e',l',z',K',\delta',P^{e'}|e,l,z,K,\delta,P^e}[(1-\theta)v_W(\omega', e', l'; z', K', \delta', P^{e'})^{1-\alpha} + \theta v_R(\omega', e', l'; z', K', P^{e'}, \delta')^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$
   where v_j are the value functions, obtained in step 2. In this step, an additional state variable is included explicitly: P^e.

   (c) Depending on the realization for z' and d', compute the joint distribution of wealth, labor efficiency and employment status.

   (d) To generate a long time series of the movement of the economy, repeat substeps b) and c).

4. Use the time series from step 2 and perform a regression of lnK' and P^e on constants and lnK, for all possible values of z and d. This way, the new aggregate laws of motion are obtained.

5. Make a comparison of the laws of motion from step 4 and step 1. If they are almost identical and their predictive power is sufficiently good, the solution algorithm is completed. If not, make a new guess for the laws of motion, based on the linear combination of laws from steps 1 and 4. Then, proceed to step 2.

10 Appendix D. Quantitative model: $\tau^{lb} = 0$

In this simple extension, I consider the case in which $\tau^{lb} = 0$. This is conducted so that the effect of the changing τ^{lb}, which inevitably comes as a consequence of changing C_f, does not confound the analysis of the changing leverage. For example, when C_f increases, the government will collect more tax revenue than before in the good aggregate states, and less revenue in bad aggregate states (compared with the case with low C_f). This means that labor taxes τ^{lb} will be higher in the good aggregate states and lower in the bad aggregate states (compared with the case with low C_f). The poorer households lose more, with the caveat that the second quintile loses the most. This can be explained by the fact that the poorest households (in the first quintile) do not have much savings, while it is the households in the second quintile that save the most in the safe asset.

Table 12: $\tau^{lb} = 0$: Long-run results

Moments	$C^f = 0.0$	$C^f = 0.3$
Welfare	37.88	38.28
Wealth GINI	0.5959	0.5943
$E(K)$	5.765	5.762
Pretax E(r^b)	2.24	1.77
Pretax E(r^s)	2.36	2.61
Post-tax E(r^{bP})	1.70	1.84
Post-tax E(r^{sP})	1.79	1.93
Post-tax E($r^{sP} - r^{bP}$)	0.094	0.092

Table 13: $\tau^{lb} = 0$: Percent changes in welfare when moving from $C_f = 0.3$ to $C_f = 0$

	Welfare change				
Quintile	1	2	3	4	5
% change	-0.30	-0.40	-0.30	-0.27	-0.26