

Rational Expectations and
Ambiguity: A Comment on Abel
(2002)

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Abstract

Abel (2002) proposes a resolution of the *riskfree rate* and the *equity premium puzzles* by considering *pessimism* and *doubt*. Pessimism is characterized by subjective probabilistic beliefs about asset returns that are stochastically dominated by the objective distribution of these returns. The subjective distribution is characterized by doubt if it is a mean-preserving spread of the objective distribution. This note offers a decision theoretic foundation of Abel's ad-hoc definitions of pessimism and doubt under the assumption that individuals exhibit *ambiguity attitudes* in the sense of Schmeidler (1989). In particular, we show that the behavior of a representative agent, who resolves her uncertainty with respect to the true distribution of asset returns in a pessimistic way, is the equivalent to pessimism in Abel's sense. Furthermore, a representative agent, who takes into account pessimistic as well as optimistic considerations, may result in the equivalent to doubt in Abel's sense.

Keywords: rational expectations, ambiguity, Choquet expected utility, pessimism, optimism, equity premium puzzle, riskfree rate puzzle

JEL Classification Numbers: D81, G20.

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1 Introduction

Abel (2002) convincingly argues that the assumption of *pessimism* and *doubt* may both help to resolve the *riskfree rate puzzle* (Weil, 1989) and the *equity premium puzzle* (Mehra and Prescott, 1985). By dropping the *rational expectations* assumption, Abel defines a *pessimist* as a decision maker whose subjective probabilistic belief about asset returns is stochastically dominated by the objective distribution of these returns. Accordingly, a decision maker is characterized by *doubt* if her subjective probabilistic belief about asset returns represents a mean-preserving spread of the objective distribution. As a shortcoming of his approach, Abel does not provide any further explanation why individuals might systematically commit such a specific violation of the *rational expectations* assumption.

The present note offers a decision-theoretic rationale for the occurrence of decision making that can be formally described as pessimism or doubt in the sense of Abel. Key to our approach is the assumption that individuals may exhibit *ambiguity attitudes* in the sense of Schmeidler (1989) and who may thus, for example, commit the Ellsberg Paradox (Ellsberg, 1961). Following Schmeidler, we formalize such individuals as CEU (=Choquet Expected Utility) decision makers, that is, they maximize expected utility with respect to *non-additive beliefs*. Properties of non-additive beliefs are used in the literature for formal definitions of, e.g., *ambiguity* and *uncertainty attitudes* (Schmeidler, 1987; Epstein, 1999; Ghirardato and Marinacchi, 2002), *pessimism* and *optimism* (Eichberger and Kelsey, 1999; Wakker, 2001; Chateauneuf et al., 2004), as well as *sensitivity to changes in likelihood* (Wakker, 2004).

Our approach focuses on non-additive beliefs that are defined as *neo-additive capacities*¹ in the sense of Chateauneuf et al. (2004). Neo-additive capacities are non-additive beliefs that stand for marginal deviations from additive beliefs such that uncertainty is resolved by a combination of pessimistic and optimistic attitudes. In particular, a neo-additive capacity is characterized by a parameter δ (*degree of ambiguity*) which measures the lack of confidence the decision maker has in some additive probability distribution π . Moreover, the ambiguous part of a decision maker's belief puts some weight (measured by the *degree of optimism* λ) on the best consequence as well as some weight (measured by the *degree of pessimism* $\gamma = 1 - \lambda$) on the worst consequence possible.

In the context of Abel's model, we interpret this additive probability distribution π as the representative agent's estimator for the underlying objective probability process of asset returns. Under the *rational expectations* paradigm the estimator π must, first, coincide with the "true" probability distribution and, second, the individual must not be ambiguous about her subjective belief, i.e., $\delta = 0$. Analogously to the rational expectations approach, we assume that π is indeed the correct estimator for the "true" probability distribution. However, our

¹"neo" stands here for "non-linear on extreme outcomes".

approach deviates from the rational expectations paradigm since we allow for the possibility that the decision maker is not entirely certain about whether her estimator π coincides with the "true" probability distribution. Hence, $\delta > 0$ might be possible. The predominantly pessimistic (optimistic) CEU decision maker of our model then resolves this lack of confidence in her estimator π in a pessimistic (optimistic) way by putting additional decision-weight on the possibility that the worst (best) consequence realizes for which $\gamma = 1$ ($\lambda = 1$).

Since the assumption of CEU decision makers with purely pessimistic beliefs successfully accommodates widely observed paradoxes of the Ellsberg type, our results support the presumption that real-life individuals can be formally described as pessimistic decision makers in the model of Abel. Even more relevantly, our decision theoretic foundation of Abel's assumption of doubt is related to recent empirical evidence showing that real-life decision makers take into account optimistic as well as pessimistic considerations (Kilka and Weber, 2001; Abdellaoui et al., 2004; Wakker, 2004).

Chen and Epstein (2002) also critically discuss Abel's ad hoc assumptions and propose instead to consider ambiguity averse decision makers defined according to the multiple-priors model of Gilboa and Schmeidler (1989). While our motivation is similar to Chen and Epstein, our approach differs in two important respects. First, while the multiple-priors approach can be used to equivalently describe pessimistic decision behavior in the sense of Abel, it cannot provide a formal equivalent for Abel's notion of doubt since the multiple-priors model neglects any optimistic considerations in the case of ambiguity. Second, our assumption of neo-additive capacities where the subjective estimator π just coincides with the true probability distribution represents only a slight - though in our opinion compelling - interpretational deviation from the rational expectations assumption.

The remainder of this note proceeds as follows. Section 2 introduces the reader to Choquet expected utility theory with a strong focus on neo-additive capacities. In Section 3 we demonstrate that a CEU decision maker with purely pessimistic beliefs can equivalently be formalized as a pessimist in the sense of Able. We also show that CEU decision making which takes into account optimistic as well as pessimistic considerations is the analogue to doubt in Abel's sense.

2 Choquet Expected Utility Theory and Neo-Additive Capacities

As a proposal for accommodating the Ellsberg paradox (Ellsberg, 1961), CEU theory was first axiomatized by Schmeidler (1986, 1989) for the framework of Anscombe and Aumann (1963) who assume the existence of random devices, generating objective probabilities. Subsequently, Gilboa (1987) as well as Sarin and Wakker (1992) have presented CEU axiomatizations for the Savage (1954) frame-

work - where probabilities are derived from betting behavior as an exclusively personalistic concept - whereby Sarin and Wakker (1992) additionally assume the existence of *ambiguous* versus *unambiguous* events. CEU theory is equivalent to *cumulative prospect theory* (Tversky and Kahneman, 1992; Wakker and Tversky, 1993) restricted to the domain of gains (compare Tversky and Wakker, 1995). Moreover, as a representation of preferences over lotteries CEU theory coincides with *rank dependent utility theory* as introduced by Quiggin (1981, 1982), which is used to accommodate Allais-paradoxes (Allais, 1954).

Adopting the Anscombe-Aumann framework, we presume that the set of *consequences*, X , is some set of *lotteries* (=objective probability distributions). An *act*, f , is then a mapping from the set of states of the world into some set of *consequences*, i.e., $f : S \rightarrow X$. Given that preferences over acts satisfy the Schmeidler axioms, such preferences are representable by utility numbers that result from (Choquet-) integration of vonNeumann-Morgenstern utility indices $u : X \rightarrow \mathbb{R}$ with respect to some *capacity*. A *capacity* (non-additive belief), ν , on the state space S is a real-valued set function on the subsets of S which satisfies

- (i) $\nu(\emptyset) = 0, \nu(S) = 1$
- (ii) $A \subset B \Rightarrow \nu(A) \leq \nu(B)$

For $A \subset S$ let $u(f(A)) := u(f(s))$ if $u(f(s)) = u(f(s'))$ for all $s, s' \in A$. For a given act f denote by A_1, \dots, A_m the partition of S such that $u(f(A_1)) > \dots > u(f(A_m))$. Define

$$w(A_i) := [\nu(A_1 \cup \dots \cup A_i) - \nu(A_1 \cup \dots \cup A_{i-1})], \quad (1)$$

where we apply the convention that $\nu(A_1 \cup \dots \cup A_0) = 0$. Recall the definition of Choquet integration:

Definition 1: *The Choquet expected utility of an act f with respect to capacity ν is defined by*

$$\mathbb{C}EU(f, \nu) := \sum_{i=1}^m u(f(A_i)) \cdot w(A_i) \quad (2)$$

Definition 2 (Chateauneuf et al., 2004): **Neo-additive capacities**

A neo-additive capacity ν is defined as a linear combination of (i) an additive belief π , (ii) a non-additive belief ω^p (where only the universal event S is considered as relevant), and (iii) a non-additive belief ω^o (where only the null event \emptyset is considered as irrelevant). Formally:

$$\nu(A) := (1 - \delta) \cdot \pi(A) + \delta(\lambda \cdot \omega^o(A) + \gamma \cdot \omega^p(A))$$

with $\delta \in (0, 1]$, $\lambda, \gamma \in [0, 1]$ such that $\lambda + \gamma = 1$, and

$$\omega^o(A) = 1 \text{ if } A \neq \emptyset$$

$$\omega^o(A) = 0 \text{ if } A = \emptyset$$

$$\omega^p(A) = 0 \text{ if } A \subset S$$

$$\omega^p(A) = 1 \text{ if } A = S$$

The CEU of an act f with respect to a neo-additive capacity ν is given by:

$$\begin{aligned} \text{CEU}(f, \nu) &= (1 - \delta) \cdot \sum_{i=1}^m \pi(A_i) \cdot u(f(A_i)) \\ &+ \delta \cdot \left(\lambda \cdot \max_{s \in S} u(f(s)) + \gamma \cdot \min_{s \in S} u(f(s)) \right). \end{aligned}$$

We refer to the parameter δ as the decision maker's *degree of ambiguity* since it has a straightforward interpretation as a measure of how confidently the individual believes that the additive measure π indeed reflects the true probability distribution of an underlying random process. The individual's ambiguity about the additive measure π is then resolved for neo-additive capacities by focussing on the extreme outcomes $\max_{s \in S} u(f(s))$ and $\min_{s \in S} u(f(s))$. How much an ambiguous individual cares about the best (worst) outcome possible for a chosen act is determined by her *degree of optimism* $\lambda \in [0, 1]$ (*degree of pessimism* $\gamma \in [0, 1]$). For example, if $\gamma = 1$ ($\lambda = 1$) we speak of a *purely pessimistic* (*optimistic*) decision maker since her ambiguity about the true probability leads her to particularly focus on the worst (best) consequence associated with her possible choices.

Remark. Notice that purely optimistic ($\lambda = 1$), respectively pessimistic ($\gamma = 1$), neo-additive capacities are *concave*, respectively *convex*, capacities. CEU decision makers with optimistic, respectively pessimistic, beliefs are therefore *ambiguity prone*, respectively *averse*, in the sense of Schmeidler's (1989) definition of ambiguity attitudes. As a consequence, CEU decision makers with purely pessimistic neo-additive capacities may commit the two-urn paradox as described in Ellsberg (1961), which violates the assumption that individuals actually decide under uncertainty as if they assigned some additive probability measure to events. More recent investigations (Kilka and Weber, 2004; Abdellaoui et al., 2004; Wakker, 2004) suggest that, besides expressing ambiguity aversion, most decision makers overweight the relevance of rather unlikely events so that a corresponding probability weighting function would be inversely S-shaped. Such a decision behavior can be well captured by CEU with respect to neo-additive capacities such that $0 < \gamma, \lambda$ and $\lambda \leq \gamma$.

3 A Decision Theoretic Foundation of Abel's Pessimism and Doubt

The representative individual of Abel's (2002) model (cf. also Lucas, 1978) holds some asset which produces returns $r = \ln R \in \mathbb{R}$ according to some objectively given probability distribution π . r denotes the net rate of return and R the gross rate of return of the underlying asset. Suppose that this asset may produce m different returns, so that we can assume some finite partition A_1, \dots, A_m of the state space S whereby greater indices of the events indicate lower returns, i.e., $r(A_j) > r(A_{j+1})$ for $j \in \{1, \dots, m-1\}$.

In his proposal for a resolution of the *riskfree-rate* and the *equity premium puzzles*, Abel exploits the difference between the expected utility of the asset-returns with respect to the objective probability distribution, $\sum_{i=1}^k u(A_i) \cdot \pi(A_i)$, and the according expected utility of the asset-returns with respect to some subjective probability distribution π^* , i.e., $\sum_{i=1}^k u(A_i) \cdot \pi^*(A_i)$. Abel defines a pessimist as follows:

Definition 3 (Abel, 2002): *A decision maker is a pessimist in the sense of Abel, if and only if, her subjective probability distribution π^* over asset-returns is (strictly) first-order stochastically dominated by the objective probability distribution π , i.e., for all $k \in \{1, \dots, m\}$,*

$$\sum_{i=1}^k \pi^*(A_i) \leq \sum_{i=1}^k \pi(A_i)$$

and for some $k \in \{1, \dots, m\}$,

$$\sum_{i=1}^k \pi^*(A_i) < \sum_{i=1}^k \pi(A_i)$$

Doubt in the sense of Abel is defined as follows:

Definition 4 (Abel, 2002): *A decision maker is an individual with doubt in the sense of Abel, if and only if, her subjective probability distribution π^* over asset-returns represents a mean-preserving spread of the objective probability distribution π , i.e.,*

$$\mathbb{E}^*(r) = \sum_{i=1}^k \pi^*(A_i) \cdot r(A_i) = \sum_{i=1}^m \pi(A_i) \cdot r(A_i) = \mathbb{E}(r)$$

and

$$\text{var}^*(r) = \mathbb{E}^*(r - \mathbb{E}^*(r))^2 > \mathbb{E}(r - \mathbb{E}(r))^2 = \text{var}(r)$$

Observe that the only relevant act in Abel's model is *holding the asset*, so that a CEU decision maker with non-additive belief ν evaluates the asset as $\sum_{i=1}^k u(A_i) \cdot w(A_i)$ where $w(A_i)$ is given by (1).

We now show that our definition of a purely pessimistic CEU decision maker can be considered as a formal special case of Abel's definition.

Proposition 1: *A representative agent CEU decision maker with neo-additive capacity ν such that $\gamma = 1$ can be equivalently characterized as a pessimist in the sense of Abel whereby the subjective probability distribution π^* is defined as follows:*

$$\begin{aligned}\pi_i^* & : = (1 - \delta) \cdot \pi(A_i) \text{ for } i \in \{2, \dots, m - 1\}, \text{ and} \\ \pi_m^* & : = (1 - \delta) \cdot \pi(A_m) + \delta\end{aligned}$$

Proof: Notice that (1) implies for purely pessimistic beliefs, i.e., $\gamma = 1$,

$$w_i = (1 - \delta) \cdot \pi(A_i) \text{ for } i \in \{1, \dots, m - 1\}$$

and

$$w_m = (1 - \delta) \cdot \pi(A_m) + \delta$$

whereby the last equation can be equivalently written as

$$w_m = 1 - (1 - \delta) \cdot \sum_{i=1}^{m-1} \pi(A_i)$$

Now define $\pi_i^* := w_i$ for $i \in \{1, \dots, m\}$, so that a CEU decision maker with neo-additive capacity ν evaluates the asset **as if** she was an expected utility maximizer with subjective (additive) belief π^* . Moreover, observe that

$$\sum_{i=1}^k \pi_i^*(A_i) = (1 - \delta) \cdot \sum_{i=1}^k \pi(A_i) < \sum_{i=1}^k \pi(A_i) \text{ for } k \in \{1, \dots, m - 1\}$$

and

$$\sum_{i=1}^m \pi_i^*(A_i) = \sum_{i=1}^m \pi(A_i) = 1$$

Thus, the accordingly defined subjective pessimistic probability distribution π^* is (strictly) first-order stochastically dominated by the objective probability distribution π . This proves our claim. \square

We next demonstrate that a CEU decision maker might evaluate the asset in Abel's model **as if** she was an expected utility maximizer with subjective (additive) belief π^* where π^* is a mean-preserving spread of the true distribution π .

Proposition 2: *Consider a representative agent CEU decision maker with neo-additive capacity ν such that*

$$\mathbb{E}(r) = \lambda \cdot r(A_1) + \gamma \cdot r(A_m) \quad (3)$$

and $\pi(A_1) + \pi(A_m) < 1$. Such a CEU decision maker can be equivalently characterized as an individual with doubt in the sense of Abel whereby the subjective probability distribution π^* is defined as follows:

$$\begin{aligned} \pi_i^* & : = (1 - \delta) \cdot \pi(A_i) \text{ for } i \in \{2, \dots, m-1\}, \text{ and} \\ \pi_1^* & : = (1 - \delta) \cdot \pi(A_1) + \delta \cdot \lambda \\ \pi_m^* & : = (1 - \delta) \cdot \pi(A_m) + \delta \cdot \gamma \end{aligned}$$

Proof: At first notice that assumption (3) entails

$$\begin{aligned} \mathbb{E}^*(r) & = \sum_{i=1}^k (1 - \delta) \cdot \pi(A_i) \cdot r(A_i) + \delta (\lambda \cdot r(A_1) + \gamma \cdot r(A_m)) \\ & = (1 - \delta) \cdot \mathbb{E}(r) + \delta \cdot \mathbb{E}(r) = \mathbb{E}(r), \end{aligned}$$

i.e., π^* and π have identical mean. Now turn to the variances:

$$\begin{aligned} \text{var}^*(r) & = \sum_{i=1}^m (1 - \delta) \cdot \pi(A_i) \cdot [r(A_i) - \mathbb{E}(r)]^2 \\ & \quad + \delta \cdot \lambda \cdot [r(A_1) - \mathbb{E}(r)]^2 + \delta \cdot \gamma \cdot [r(A_m) - \mathbb{E}(r)]^2 \\ & = (1 - \delta) \cdot \text{var}(r) + \delta (\lambda \cdot [r(A_1) - \mathbb{E}(r)]^2 + \gamma \cdot [r(A_m) - \mathbb{E}(r)]^2) \end{aligned} \quad (4)$$

Since

$$\begin{aligned} & \lambda \cdot [r(A_1) - \mathbb{E}(r)]^2 + \gamma \cdot [r(A_m) - \mathbb{E}(r)]^2 \\ & > \pi(A_1) \cdot [r(A_1) - \mathbb{E}(r)]^2 + \dots + \pi(A_m) \cdot [r(A_m) - \mathbb{E}(r)]^2 \\ & = \text{var}(r) \end{aligned}$$

whenever assumption (3) holds and $\pi(A_1) + \pi(A_m) < 1$, equation (4) gives the desired result

$$\text{var}^*(r) > \text{var}(r),$$

i.e., the subjective probability distribution π^* is a mean-preserving spread of π . \square

Remark. If the distribution of returns is symmetric, i.e., if R is log-normal and therefore $r = \ln R$ is normal as assumed by Abel (2002), then assumption (3) holds iff $\lambda = \gamma = 0.5$, since, under symmetry, $r(A_1) - \mathbb{E}(r) = \mathbb{E}(r) - r(A_m)$.

Remark. The above results are established under the assumption that the CEU decision maker is the representative agent of the economy. An alternative way to read our results in Proposition 2 is to assume an economy that is populated by a proportion λ of purely optimistic decision makers and a proportion $\gamma = 1 - \lambda$ of purely pessimistic decision makers. Analogously the parameters δ , λ and γ can themselves be regarded as averages over heterogenous agents.

References

- Abdellaoui, M., Vossman, F., and M. Weber (2004), "Choice-based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty", *mimeo*
- Abel, A.B. (2002), "An Exploration of the Effects of Pessimism and Doubt on Asset Returns", *Journal of Economic Dynamics and Control* **26**, 1075-1092.
- Allais, M. (1953), "Le Comportement de l'Homme Rationnel devant le Risque: Critique des postulats et axiomes de l'École Américaine", *Econometrica* **21**, 503-546. Reprinted in English as part II in Allais, M., and O. Hagen [eds.] (1979), *Expected Utility Hypotheses and the Allais Paradox*, D. Reidel: Dordrecht etc.
- Anscombe, F.J, and R.J. Aumann (1963), "A Definition of Subjective Probability", *Annals of American Statistics* **34**, 199-205.
- Chateauneuf, A., Eichberger, J., and S. Grant (2004), "Choice under Uncertainty with the Best and Worst in Mind: Neo-additive Capacities", *mimeo*
- Chen, Z., and L.G. Epstein (2002), "Ambiguity, Risk and Asset Returns in Continuous Time", *Econometrica* **70**, 1403-1443.
- Eichberger, J., and D. Kelsey (1999), "E-Capacities and the Ellsberg Paradox", *Theory and Decision* **46**, 107-140.
- Ellsberg, D. (1961), "Risk, Ambiguity and the Savage Axioms", *Quarterly Journal of Economics* **75**, 643-669.
- Epstein, L.G. (1999), "A Definition of Uncertainty Aversion", *The Review of Economic Studies* **66**, 579-608.
- Ghirardato, P., and M. Marinacci (2002), "Ambiguity Made Precise: A Comparative Foundation", *Journal of Economic Theory* **102**, 251-289
- Gilboa, I. (1987), "Expected Utility with Purely Subjective Non-Additive Probabilities", *Journal of Mathematical Economics* **16**, 65-88.
- Gilboa, I., and D. Schmeidler (1989), "Maxmin Expected Utility with Non-Unique Priors", *Journal of Mathematical Economics* **18**, 141-153.
- Kilka, M., and M. Weber (2001), "What determines the Shape of the Probability Weighting Function under Uncertainty", *Management Science* **47**, 1712-1726.
- Mehra, R., and E.C. Prescott (1985), "The Equity Premium: A Puzzle", *Journal of Monetary Economics* **15**, 145-161.

- Quiggin, J.P. (1981), "Risk Perception and Risk Aversion among Australian Farmers", *Australian Journal of Agricultural Economics* **25**, 160-169.
- Quiggin, J.P. (1982), "A Theory of Anticipated Utility", *Journal of Economic Behavior and Organization* **3**, 323-343.
- Sarin, R., and P.P. Wakker (1992), "A Simple Axiomatization of Nonadditive Expected Utility", *Econometrica* **60**, 1255-1272.
- Savage, L.J. (1954). *The Foundations of Statistics*, John Wiley and Sons, Inc.: New York, London, Sydney.
- Schmeidler, D. (1986), "Integral Representation without Additivity", *Proceedings of the American Mathematical Society* **97**, 255-261.
- Schmeidler, D. (1989), "Subjective Probability and Expected Utility without Additivity", *Econometrica* **57**, 571-587.
- Tversky, A., and D. Kahneman (1992), "Advances in Prospect Theory: Cumulative Representations of Uncertainty", *Journal of Risk and Uncertainty* **5**, 297-323.
- Tversky, A., and P.P. Wakker (1995), "Risk Attitudes and Decision Weights", *Econometrica* **63**, 1255-1280.
- Wakker, P.P. (2001), "Testing and Characterizing Properties of Nonadditive Measures through Violations of the Sure-Thing Principle", *Econometrica* **69**, 1039-1059.
- Wakker, P.P. (2004), "On the Composition of Risk Preference and Belief," *Psychological Review* **111**, 236-241.
- Wakker, P.P, and A. Tversky (1993), "An Axiomatization of Cumulative Prospect Theory", *Journal of Risk and Uncertainty* **7**, 147-176.
- Weil, P. (1989), "The Equity Premium Puzzle and the Riskfree Rate Puzzle", *Journal of Monetary Economics* **24**, 401-421.

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